TSUKUBA J. MATH.
Vol. 14 No. 2 (1990), 435—436

A REMARK ON R. POL'S THEOREM COMCERNING A-WEAKLY INFINITE-DIMENSIONAL SPACES

By

Zhu JIAN-PING

For notations and relevant definitions we refer to [1].

THEOREM (MA). *There is no universal space in the class of all metrizable separable A-weakly infinite-dimensional spaces.*

R. Pol proved this theorem in [1] under CH. The proof we shall give is similar with the one given in [1] but a little more direct.

LEMMA 1. *Let* $S\subset I^{\omega}$ *be a countable union of zero-dimensional subsets. If* $C\subset I^{\omega}$ *satisfies that for any open neighbourhood* U *of* S $|C\setminus U|<\mathfrak{c}$, *then* $C\cup S$ *is A-weakly infinite-dimensional.*

The proof is parallel to the proof of Lemma 1 in [1], noting that in I^{ω} every subset with cardinality less than $\mathfrak{c}$ is zero-dimensional.

LEMMA 2 (MA). *Let* $\{G_{\alpha}:\alpha<\lambda\}$ *be a family of open neighbourhoods of* Σ *in* I^{ω} *and* $\lambda<\mathfrak{c}$, *where* $\Sigma=\{x\in I^{\omega}$: *all but finitely many coordinates of* x *are equal to zero*$\}$. *Then there exist positive numbers* $a_i\in I(i\in\omega)$ *such that* $\bigcup_i[0, a_i]\subset\bigcap\{G_{\alpha}:\alpha<\lambda\}$. *Therefore, if* $E\subset I^{\omega}$ *can be embedded in an A-weakly infinite-dimensional space, then* $\bigcap\{G_{\alpha}:\alpha<\lambda\}\setminus E\neq 0$.

PROOF. Let $\mathcal{B}=\{[0, 1/n]:n>0\}$. We define $\boldsymbol{P}=\{(a, b)$: a is a finite sequence in $\mathcal{B}$ & $b\in[\lambda]^{<\omega}\}$ and for any $(a', b'), (a, b)\in\boldsymbol{P}$, where $a=(I_0, I_1, \cdots, I_n)$ and $a'=(I'_0, I'_1, \cdots, I'_{n'})$, $(a', b')\leqq(a, b)$ iff $b'\supset b$, $n\leqq n'$, $I_i=I'_i$ for any $i\leqq n$ and if $n<n'$, $\prod_{i\leqq n'}I'_i\times\prod_{i>n'}I\subset\bigcap\{G_{\alpha}:\alpha\in b\}$. It is obvious that $\leqq$ is a partial order on $\boldsymbol{P}$. Since all of first components of elements of $\boldsymbol{P}$ are countable, $\boldsymbol{P}$ is *ccc* (in fact σ-centred).

Let $D_{\alpha}=\{(a, b)\in\boldsymbol{P}:\alpha\in b\}$ and $F_n=\{(a, b)$: the length of a is larger than $n\}$. It is easily seen that D_{α} is dense in $\boldsymbol{P}$ for any $\alpha<\lambda$. Now we want to

Received September 14, 1989. Revised November 22, 1989.