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SMASH PRODUCTS AND COMODULES OF LINEAR MAPS

By

K. -H. ULBRICH*

Let G be a finite group and A be a G-graded algebra over a commutative ring k. Consider the G-graded right A-module $U=\bigoplus_{\sigma\in G}A(\sigma)$ where $A(\sigma)=A$ has grading shifted by σ. Năstăsescu and Rodinò [5] proved that

(1) $$\mathrm{End}_{A\text{-}gr}(U)*G\cong\mathrm{End}_{A}(U),\quad\text{and}\quad A\#k[G]^{*}\cong\mathrm{End}_{A\text{-}gr}(U)$$

where $\mathrm{End}_{A\text{-}gr}(U)$ denotes the algebra of graded A-endomorphisms of U, and $*$ means crossed product, [5], Theorems 1.2 and 1.3. The proofs are given by some explicit matrix computations relying on a graded isomorphism $\mathrm{End}_{A}(U)\cong M_{n}(A)$, $n=|G|$, [5], Prop. 1.1. The first isomorphism of (1) has recently been generalized to

(2) $$\mathrm{End}_{A\text{-}gr}(U)*G\cong\mathrm{END}_{A}(U),\qquad [2],\ \text{Thm. } 3.3,$$

for not necessarily finite groups G. The purpose of this paper is to give Hopf algebraic versions of (1) and (2). Write $H=k[G]$. First note that the above crossed products are also smash products. Furthermore, a G-graded k-module is the same as an H-comodule, and the A-isomorphism

$$U\xrightarrow{\ \sim\ }H\otimes A,\quad a(\sigma)\longmapsto\sigma^{-1}\otimes a(\sigma),\qquad a(\sigma)\in A(\sigma),$$

is H-colinear where $H\otimes A$ has coaction $\alpha: H\otimes A\to H\otimes A\otimes H$ defined by

(3) $$\alpha(h\otimes a)=\sum h_{(1)}\otimes a_{(0)}\otimes h_{(2)}a_{(1)},\qquad h\in H,\ a\in A.$$

Now let H be any Hopf algebra over k and set $U=H\otimes A$ for a right H-comodule algebra A. Let $\mathrm{End}_{A}^{H}(U)$ be the algebra of right A-linear maps $U\to U$ which are colinear with respect to (3). We shall generalize (1), for H finite over k, to

(4) $$\mathrm{End}_{A}^{H}(U)\#H\cong\mathrm{End}_{A}(U)\quad\text{and}\quad A\#H^{*}\cong\mathrm{End}_{A}^{H}(U).$$

It was pointed out in [5] that (1) implies the duality theorems of Cohen and Montgomery [4]. Correspondingly, (4) may be viewed as an improvement of the duality result for finite Hopf algebras [3], Cor. 2.7. Note that the second

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