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COMPLETE SPACE-LIKE HYPERSURFACES OF A DE SPITTER SPACE WITH CONSTANT MEAN CURVATURE

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Introduction

Let $M_s^m(c)$ be an m-dimensional connected semi-Riemannian manifold of index s and of constant curvature c, which is called an *indefinite space form of index* s or simply a *space form* according as $s>0$ or $s=0$. An m-dimensional space form of constant curvature c is only denoted by $M^m(c)$. The study of hypersurfaces with constant mean curvature of $M^{n+1}(c)$ was initiated by Nomizu and Smyth [13], who proved some excellent results.

It is seen that a complete space-like hypersurface of a Minkowski space R_1^{n+1} possesses a remarkable Bernstein property in the maximal case by Calabi [3] and Cheng and Yau [5]. As a generalization of the Bernstein type problem a complete space-like maximal submanifold M of $M_p^{n+p}(c)$ was recetly characterized by Ishihara [9] under a certain condition. In particular, it is proved that if c is non-negative, then M it totally geodesic.

On the other hand, it is pointed out by Marsden and Tipler [10] that space-like hypersurfaces with constant mean curvature of arbitrary spacetimes have interest in relativity theory. An entire space-like hypersurface with constant mean curvature of a Minkowski space is investigated by Goddard [8] and Treibergs [19]. It is well known as standard models of space-like hypersurfaces with constant mean curvature of a Minkowski space R_1^{n+1} (resp. a de Sitter space $S_1^{n+1}(c)$) that we have hyperboloids $H^k(c)\times R^{n-k}$ (resp. $H^k(c_1)\times S^{n-k}(c_2)$ and R^n), where $k=0, 1, \cdots, n$. After some perturbations conserving constant mean curvatures, Goddard [8] conjectured the following two results: the only space-like hypersurfaces with constant mean curvature of R_1^4 are the hyperboloids and three classes of space-like hypersurfaces $S^3(c_2)$, R^3 and $H^3(c_1)$ are the only complete space-like hypersurfaces with constant mean curvature which exist in $S_1^4(c)$. Stumbles [18] and Treibergs [19] however constructed many entire such hypersurfaces of R_1^{n+1} different from the hyperboloids.

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