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SPAN ZERO CONTINUA AND THE PSEUDO-ARC

Dedicated to Professor Ryosuke Nakagawa on his 60th birthday

By

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0. Introduction

A compact connected metric space is called a *continuum*. Let X be a continuum and d be a metric of X. A. Lelek [6], [7] defined the *span, semispan, surjective span* and *surjective semispan* by the following formulas (the map π_j denotes the projection map from $X\times X$ onto the i-th factor).

$\tau=\sigma,\ \sigma_0,\ \sigma^*,\ \sigma_0^*.$

$$\tau=\sup\left\{ c\geqq 0 \left| \begin{array}{l} \text{there exists a continuum } Z\subset X\times X \text{ such that} \\ Z \text{ satisfies the condition } \tau) \text{ and} \\ d(x, y)\geqq c \quad \text{for each } (x, y)\in Z \end{array} \right. \right\}.$$

Where the condition τ) is

$$\begin{array}{ll} \pi_1(Z)=\pi_2(Z) & \text{if } \tau=\sigma \\ \pi_1(Z)\supset\pi_2(Z) & \text{if } \tau=\sigma_0 \\ \pi_1(Z)=\pi_2(Z)=X & \text{if } \sigma=\sigma^* \\ \pi_1(Z)=X & \text{if } \tau=\sigma_0^* \end{array}$$

The property of having zero span (semispan, surjective span, surjective semispan resp.) does not depend on the choice of metrics of X.

A continuum is said to be *arc-like* if it is represented as the limit of an inverse sequence of arcs. It is known that each arc-like continuum has span zero. But it is not known whether the converse implication is true or not. A continuum X is said to be *hereditarily indecomposable* if each subcontinuum Y of X cannot be represented as the union of two proper subcontinua of Y. Hereditarily indecomposable arc-like continuum is topologically unique. It is called the *pseudo-arc* and denoted by P in this paper. It is known to be a homogeneous plane continuum and is also important in span theory. For example, each span zero continuum is a continuous image of the pseudo-arc ([11] and [2]).

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