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SPECIAL ALGEBRAIC PROPERTIES OF KÄHLER ALGEBRAS

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Homogeneous Kähler manifolds M are frequently investigated via Kähler algebras $(\boldsymbol{g}, \boldsymbol{k}, j, \rho)$, where $\boldsymbol{g}$ denotes a Lie algebra of infinitesimal automorphisms of M and $\boldsymbol{k}$ the isotropy subalgebra of some point in M. Moreover, j corresponds to the complex structure tensor and ρ to the Kähler form. In particular, Kähler algebras have been used intensively in the proof of the geometric Fundamental Conjecture for homogeneous Kähler manifolds: Every homogeneous Kähler manifold is a holomorphic fiber bundle over a homogeneous bounded domain in which the fiber is (with the induced Kähler metric) the product of a flat homogeneous Kähler manifold and a compact simply connected homogeneous Kähler manifold.

Two additional properties of Kähler algebras have proven to be particularly useful. One is that $\boldsymbol{g}$ or ad $\boldsymbol{g}$ is an algebraic Lie algebra. The second one is the assumption that ρ is the differential of a leftinvariant 1-form, $\rho=d\omega$. This is the case of "j-algebras". It has been investigated intensively by Gindikin, Piatetskii-Shapiro, Vinberg and others. The proof of the Fundamental Conjecture for homogenous Kähler manifolds is much shorter for j-algebras than for general Kähler algebras. This is due to some extent to the fact that one can embed a j-algebra into an algebraic j-algebra.

The purpose of this note is threefold. First we want to prove that for the Lie algebra $\boldsymbol{g}_M$ of all infinitesimal automorphisms of an arbitrary homogeneous Kähler manifold M, the Lie algebra ad $\boldsymbol{g}_M$ is algebraic. Secondly, we decompose $\boldsymbol{g}_M$ into the orthogonal sum of j-invariant subalgebras. This decomposition will be of importance for a forthcoming publication in which we give a detailed description of $\boldsymbol{k}_M$ and the Kähler form ρ. The orthogonal decomposition in question has a simple geometric interpretation. It is essentially induced by a representation of the base domain (occuring in the Fundamental Conjecture) as a Siegel domain of type three.

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