

# AN ISOMETRIC EMBEDDING OF THE COMPLEX HYPERBOLIC SPACE IN A PSEUDO-EUCLIDEAN SPACE AND ITS APPLICATION TO THE STUDY OF REAL HYPERSURFACES

By

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## 0. Introduction.

In the last years, the use of an idea of A. Ros, [11], has meant an interesting progress in the study of several families of submanifolds of the complex projective space  $CP^n$ . This idea essentially consists in considering the first standard embedding of  $CP^n$  in a certain Euclidean space  $R^N$ , and contemplating the submanifolds of  $CP^n$  in the light of that new embedding. The first standard embedding has parallel second fundamental form and makes  $CP^n$  to be a symmetric  $R$ -space in  $R^N$ .

In particular real hypersurfaces of  $CP^n$  have been analysed under this point of view, [6], [13], and new characterizations of this important class of hypersurfaces have been obtained.

In 1986, the second author and S. Montiel, [8], made a systematic study of a certain family of real hypersurfaces of the complex hyperbolic space  $CH^n$ . In the process of classification of that family they introduced new examples without parallel in  $CP^n$ . Therefore if we could get an isometric embedding of  $CH^n$  in some Euclidean space  $R^N$  provided of as good geometric properties as those of the first standard embedding of  $CP^n$ , we could try to profound in the study of real hypersurfaces in  $CH^n$ .

On the other hand fully immersed complete submanifolds of a Euclidean space with parallel second fundamental form have been totally classified by D. Ferus, [3], [4]. As a consequence his result implies that a complete irreducible (as a Riemannian manifold) submanifold which is fully immersed in an Euclidean space with parallel second fundamental form is congruent to either an hyperplane or to an irreducible symmetric  $R$ -space immersed by means of its standard embedding. Consequently we see that there exist no an isometric immersion

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