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3-DIMENSIONAL ISOTROPIC SUBMANIFOLDS OF SPHERES

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1. Introduction

In this paper, we study 3-dimensional isotropic submanifolds in spheres. The notion of an isotropic submanifold of an arbitrary Riemannian manifold was first introduced by B. O'Neill in $[O]_1$. The basic equations for isotropic submanifolds are recalled in Section 2.

Isotropic immersions of submanifolds into spheres have been studied by, amongst others, T. Itoh, H. Nakagawa, K. Ogiue and K. Sakamoto in [I], [N-I], [I-O] and [S]. Here, we will prove the two following theorems.

THEOREM 3.1. *Let $x: M \to S^n$ be a constant isotropic immersion such that* $\dim(\mathrm{im}(h)) \leqq 3$. *Then, one of the following holds:*

(a) *M is totally geodesic in S^n,*

(b) *There exists a totally geodesic S^4 in S^n, such that the image of M is an open part of a small hypersphere of S^4,*

(c) *There exists a totally geodesic S^7 in S^n, such that the image of M is congruent with an open part of $j\left(\boldsymbol{R} \times S^2\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\right)$ in S^7, where j is defined in Section 3.*

THEOREM 3.2. *Let M be a 3-dimensional, minimal, isotropic submanifold in S^n. Then, M has constant sectional curvature.*

2. Preliminaries

In this section M will always denote a 3-dimensional totally real submanifold of $S^n(1)$. We will denote the curvature tensor of M by R. The formulas of Gauss and Weingarten are given by

$$(2.1) \qquad D_X Y = \nabla_X Y + h(X, Y) \quad \text{and} \quad D_X \zeta = -A_\zeta X + \nabla^\perp_X \zeta,$$

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