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ON CERTAIN MULTIVALENTLY STARLIKE FUNCTIONS

By

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Let $A(p)$ denote the class of functions $f(z)=z^{p}+\sum_{n=p+1}^{\infty}a_{n}z^{n}$ which are analytic in the open unit disk $E=\{z: |z|<1\}$.

A function $f(z)\in A(p)$ is called p-valently starlike with respect to the origin iff

$$\mathrm{Re}\frac{zf'(z)}{f(z)}>0 \quad \text{in} \quad E,$$

Ozaki [2, Theorem 1] proved that if $f(z)\in A(1)$ and

$$1+\mathrm{Re}\frac{zf''(z)}{f'(z)}<\frac{3}{2} \quad \text{in} \quad E, \tag{1}$$

then $f(z)$ is univalent in E.

Moreover, Umezawa [6] proved that if $f(z)\in A(1)$ satisfies the condition (1), then $f(z)$ is univalent and convex in one direction in E.

Recently, R. Singh and S. Singh [4, Theorem 6] proved that if $f(z)\in A(1)$ satisfies the condition (1), then $f(z)$ is starlike in E.

Ozaki [2, Theorem 3] proved that if $f(z)\in A(p)$ and

$$1+\mathrm{Re}\frac{zf''(z)}{f'(z)}<p+\frac{1}{2} \quad \text{in} \quad E, \tag{2}$$

then $f(z)$ is p-valent in E.

It is the purpose of the present paper to prove that if $f(z)\in A(p)$ satisfies the condition (2), then $f(z)$ is p-valently starlike in E.

This is an extended result of R. Singh and S. Singh [4, Theorem 6].

In this paper, we need the following lemma.

LEMMA 1. *Let $f(z)\in A(1)$ be starlike with respect to the origin in E.*

Let $C(r, \theta)=\{f(te^{i\theta}): 0\leqq t\leqq r<1\}$ and $T(r, \theta)$ be the total variation of $\arg f(te^{i\theta})$ on $C(r, \theta)$, so that

$$T(r, \theta)=\int_{0}^{r}\left|\frac{\partial}{\partial t}\arg f(te^{i\theta})\right|dt.$$

Then we have

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