TSUKUBA J. MATH.
Vol. 14 No. 2 (1990), 247—262

GENERALIZED COHOMOLOGICAL DIMENSION OF COMPACT METRIC SPACES

Dedicated to professor Yukihiro Kodama on his 60th birthday

By

A. N. DRANISHNIKOV

§ 0. Introduction.

The notion of homological dimension was introduced by P. S. Alexandroff in the later 1920's. The further contribution to the development of homological dimension theory on compact metric spaces was made by L. S. Pontrjagin [2], K. Borsuk [3], M. F. Bokshtein [4], V. G. Boltynanski [5], E. Dyer [6], Y. Kodama [7, 8], V. I. Kuz'minov [9] and others. New achievement of the theory are surveyed in [10]. In this paper we do not consider homological dımension theory out of compact metric spaces. Moreover everywhere in this paper cohomological language is used instead of homological one, and therefore a dual notion of cohomological dimension is considered.

A compact metric space X has *cohomological dimension with respect to an abelian group G equal or less than n* (written, $c\text{-}\dim_G X \leqq n$) iff for an arbitrary closed subset $A \subset X$ and an arbitrary integer $k \geqq n$, the inclusion $A \to X$ induces an epimorphism of k-dimensional Čech cohomology groups with coefficients.

ALEXANDROFF THEOREM [11]. *For every finite-dimensional compact metric space, we hold the equation* $c\text{-}\dim_Z X = \dim X$.

As justifiably L. Rubin remarks in [13], we do not have enough information about infinite-dimensional spaces to determine a theory. In this paper a notion of generalized cohomological dimension $c\text{-}\dim_E$ for some class of spectra $\delta = \{E\}$ is introduced. For all $E \in \mathcal{E}$, those dimensions $c\text{-}\dim_E$ coincide with the covering dimension dim for finite dimensional compacta. Namely, they are distinguished in the class of infinite-dimensional compacta.

In § 1, the inequalities

$$c\text{-}\dim_Z X \leqq c\text{-}\dim_E X \leqq \dim X$$

Received March 6, 1989. Revised November 24.