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SYMMETRIC SUBMANIFOLDS AND GENERALIZED GAUSS MAPS

Dedicated to Professor Shingo Murakami on his sixtieth birthday

By

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Introduction.

Let (M, g) be the n-dimensional unit sphere of $\boldsymbol{R}^{n+1}$ and S an r-dimensional connected submanifold of (M, g). Regarding S as a submanifold of $\boldsymbol{R}^{n+1}$, we can associate the Gauss map with it. It is a smooth mapping of S to the Grassmannian manifold G_r^{n+1} of the r-dimensional linear subspaces in $\boldsymbol{R}^{n+1}$, defined as follows; $S \ni q \to T_q S \in G_r^{n+1}$. The target space G_r^{n+1} is a riemannian symmetric space with a suitable metric. If the second fundamental form of S is parallel, the Gauss map is a totally geodesic immersion by a result in Vilms [10]. Here we note that if such a submanifold S is complete, it is characterized as a symmetric submanifold, namely a submanifold preserved by the reflections with respect to all the normal spaces, and moreover the latter submanifold is analougously defined for the case that the ambient space is a riemannian symmetric space. The purpose of this paper is to extend the above result for a symmetric submanifold of a simply connected riemannian symmetric space without Euclidean factor.

We will first consider certain submanifold classes of such a riemannian symmetric space which contain the symmetric submanifolds, and then define a generalization of Gauss map for each submanifold class. The target space of this generalization is generally a pseudo-riemannian symmetric space, and moreover if the ambient riemannian symmetric space is compact, it is a compact riemannian symmetric space. We will next show that for a symmetric submanifold our generalized Gauss map is a totally geodesic immersion, and it is moreover isometric if and only if the submanifold is totally geodesic. Last we will give the list of the target spaces of the generalized Gauss maps for our considerable submanifold classes of the simply connected irreducible riemannian symmetric spaces.

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