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COALGEBRA ACTIONS ON AZUMAYA ALGEBRAS

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Introduction.

The notion of measuring actions of coalgebras on an algebra unifies the notions of algebra automorphisms, of derivations and of higher derivations. In this paper we examine such actions of a k-coalgebra C on an Azumaya k-algebra A, where k is a commutative ring. In (2.4) we show a 1-1 correspondence between the set of measurings $C\to\mathrm{End}\,A$ and the set of certain right C^*-submodules of $C^*\otimes A$. Using this result, we show a Noether-Skolem type theorem (3.1): For example, *if k is a field, then any measuring $C\to\mathrm{End}\,A$ is inner for arbitrary C and A.*

Throughout the paper we fix a commutative ring k with 1. A linear map, an algebra, a coalgebra, $\otimes$, Hom and End mean a k-linear map, a k-algebra, a k-algebra, a k-coalgebra, $\otimes_k$, Hom_k and End_k, respectively. We fix an algebra A and a coalgebra C. C^* denotes $\mathrm{Hom}(C, k)$, the dual algebra of C [**9**, Prop. 1.1.1, p. 9].

1. Preliminaries.

Let Δ, ε be the structure maps of C and write

$$\Delta(c)=\sum_{(c)} c_{(1)}\otimes c_{(2)} \qquad \text{for } c\in C.$$

The k-module $\mathrm{Hom}(C, A)$ is an algebra with the $*$-product [**9**, p. 69]. $\mathrm{Hom}(C, A)^{\times}$ denotes the group of units in $\mathrm{Hom}(C, A)$.

1.1. DEFINITION. A linear map $f: C\to\mathrm{End}\,A$ is called a *measuring*, if $a\mapsto(c\mapsto f(c)(a))$, $A\to\mathrm{Hom}(C, A)$ is an algebra map, or equivalently if

$$f(c)(1)=\varepsilon(c)1,$$

$$f(c)(ab)=\sum_{(c)} f(c_{(1)})(a)f(c_{(2)})(b)$$

for $c\in C$, $a, b\in A$ [**9**, Def. p. 138]. We denote by

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