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HARMONIC FOLIATIONS ON A COMPLEX PROJECTIVE SPACE

By

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1. Introduction.

In 1970, D. Ferus [6] gave an estimation on the codimension of a totally geodesic foliation on a sphere and a complex projective space, and successively P. Dombrowski [1] improved his results. Moreover, R. Escobales classified Riemannian foliations satisfying a certain condition on a sphere and a complex projective space in a series of his papers [2], [3], [4], [5].

On the other hand, F. Kamber and Ph. Tondeur [7], [8] studied the index of harmonic foliations with bundle-like metric on a sphere from a view point of harmonic mappings.

Recently, H. Nakagawa and R. Takagi [11] showed that any harmonic foliations on a compact Riemannian manifold of non-negative constant sectional curvature is totally geodesic if the normal plane field is minimal.

In this paper we will prove

THEOREM. *Let* $\mathbf{P}_m(\mathbf{C})$ *be a complex projective space of complex dimension* m *with the metric of constant holomorphic sectional curvature. If* $\mathcal{F}$ *is a harmonic foliation on* $\mathbf{P}_m(\mathbf{C})$ *snch that the normal plane field is minimal, then* $\mathcal{F}$ *is totally geodesic.*

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2. Preliminaries.

We first establish some basic notations and formulas in the theory of foliated Riemannian manifolds. For details, see [9], [10], [11], [13].

Let (M, g) be an n-dimentional Riemannian manifold and $\mathcal{F}$ a foliation with codimension q on M. Considering $\mathcal{F}$ as an $(n-q)$-dimensional integrable distribution on M, we denote the orthogonal distribution of $\mathcal{F}$ by $\mathcal{F}^{\perp}$, which is called the normal plane field.

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