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SEQUENTIAL POINT ESTIMATION WITH BOUNDED RISK IN A MULTIVARIATE REGRESSION MODEL

By

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For the coefficient matrix of the multivariate regression model, consider the problem of finding an estimator with asymptotically bounded risk. The paper proposes a sequential procedure resolving the problem and investigates the asymptotic properties. Also it is shown that if additional observations with the same coefficient matrix are available, then the sequential estimator is improved on by a combined procedure.

1. Introduction

Let $x_1, x_2, \cdots$ be a sequence of mutually independent random vectors, x_i having p-variate normal distribution $N_p(\xi a_i, \Sigma)$ where a_i $(r\times 1)$ is a known vector and ξ $(p\times r)$, Σ $(p\times p)$ are unknown matrices. Denote $X_n=(x_1, x_2, \cdots, x_n)$, $A_n=(a_1, a_2, \cdots, a_n)$ and $\omega=(\xi, \Sigma)$. Then X_n $(p\times n)$ has $N_{p,n}(\xi A_n; \Sigma, I_n)$, being a multivariate regression model.

For a preassigned constant $\varepsilon>0$, we consider the problem of finding an estimator $\hat{\xi}_\varepsilon$ of the coefficient matrix ξ such that

$$R(\omega, \hat{\xi}_\varepsilon)=E_\omega[n^{-1}\operatorname{tr} Q(\hat{\xi}_\varepsilon-\xi)A_nA_n'(\hat{\xi}_\varepsilon-\xi)']\leqq\varepsilon \tag{1.1}$$

for all ω, where Q $(p\times p)$ is a positive definite matrix.

Throughout the paper, let m_0 be the smallest integer $(\geqq r)$ such that $\operatorname{rank}(A_{m_0})=r$. In the case where Σ is known, for integer n $(\geqq m_0)$, MLE of ξ is given by

$$\hat{\xi}_0(n)=X_nA_n'(A_nA_n')^{-1}$$

and from Muirhead (1982),

$$\begin{aligned}R(\omega, \hat{\xi}_0(n))&=E_\omega[n^{-1}\{\operatorname{vec}(\hat{\xi}_0(n)-\xi)\}'(A_nA_n'\otimes Q)\operatorname{vec}(\hat{\xi}_0(n)-\xi)]\\&=n^{-1}\operatorname{tr}(A_nA_n'\otimes Q)\operatorname{Cov}(\operatorname{vec}\hat{\xi}_0(n))\end{aligned} \tag{1.2}$$

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