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A CLASS OF FINITE GROUPS ADMITTING CERTAIN SHARP CHARACTERS I

Dedicated to Professor Yukihiro Kodama on his 60th birthday

By

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1. Introduction.

Let G be a finite group and χ a character of G of degree n, and let L be the image of χ on G, and $L^{\#}=L\setminus\{n\}$. H.F. Blichfeldt ([1]) and P.J. Cameron —M. Kiyota ([2]) showed that the number $B(n)$ defined by $(1/|G|)\prod_{l\in L^{\#}}(n-l)$ is an integer when χ is faithful. Such (G, χ) is called a *sharp pair of type* $L^{\#}$ and χ *a sharp character* provided $B(n)=1$.

This concept generalizes that of sharply multiply—transitive permutation groups, where a rich theory of sharp pairs has been developed and many examples are known ([2], [3], [6]). Moreover, F. Blichfeldt showed that the number $B(l)$ defined by $(a(l)/|G|)\prod_{k\in L\setminus\{l\}}(l-k)$ is an algebraic integer for any $l\in L$ whenever χ may not be faithful (where $a(l)$ denotes the number of elements x of G with $\chi(x)=l$).

Recently M. Kiyota ([5]) introduced a class of pairs of finite groups and its characters which included that of sharp pairs of finite groups; a triple (G, χ, l) is called a *sharp triple of rank r* provided $B(l)$ is a unit in the ring of algebraic integers $(r=|L^{\#}|, l\in L)$. He posed the problem of determining sharp triples when a finite subset $L^{\#}$ of $\boldsymbol{C}$ is given. This problem in the case of sharp pairs has been studied by several authors (cf. [2], [6]).

In this note we will investigate sharp triples of rank 2. There are two cases: (i) the two elements of $L^{\#}$ are algebraic conjugates (cf. Theorem 1). (ii) all elements of L are integers. However, it seems difficult to classify completely even sharp pairs in the case (ii), so we will treat it under the condition that χ is irreducible or sharp (cf. Proposition 3 and Theorem 4). In particular, we will note that all sharp triples of type $\{-1, 1\}$ are sharp pairs except for two cases in the last section (cf. Theorem 5).

Our notations are largely standard (cf. [4], [2]). Throughout this note, $\mathcal{O}$

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