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ON ALMOST M-PROJECTIVES AND ALMOST M-INJECTIVES

Dedicated to Professor Tuyosi Oyama on his 60th birthday

By

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We have defined aconcept of almost M-projectives and almost M-injectives in [4] and [9], respectively. In the first section of this paper we give some relations among lifting modules, mutually almost relative projectivity and locally semi-T-nilpotency. After giving a criterion of mutually almost relative projectivity between two hollow modules in the second section, we give a characterization of lifting modules over a right artinian ring. Further we show a difference between M-projectives and almost M-projectives. Those dual properties are gives in the third and fourth sections with sketch of proofs.

We shall give several characterizations of right Nakayama (resp. right co-Nakayama) rings in terms of almost relative projectives (resp. almost relative injectives) in forthcoming papers (cf. [9]).

1. Almost projectives.

Throughout this paper R is an associative ring with identity. Every module M is a unitary right R-module. Let M be an R-module and K a submodule of M. If $M\neq M'+K$ for any proper submodule M' of M, then K is called a *small submodule* in M. If $K\cap K'\neq 0$ for every non-zero submodule K' of M, we say that K is an *essential submodule* of M. If every proper submodule of M is always small in M, M is called a *hollow module* and we dually call M a *uniform module*, provided every non-zero submodule is essential in M. If $\mathrm{End}_R(M)$, the ring of endomorphisms of M, is a local ring, M is called an *le module*. By $J(M)$ and $\mathrm{Soc}(M)$ we denote the *Jacobson radical* and the *socle* of M, respectively and $|M|$ is the *length* of M.

Following K. Oshiro [15] and [16] we define a lifting (resp. extending) module. If for any submodule N of M, there exists a direct decomposition $M=M_1\oplus M_2$ such that

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