

A CHARACTERIZATION OF A REAL HYPERSURFACE OF TYPE B

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Introduction.

A complex n -dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form is a complex projective space $P_n\mathbf{C}$, a complex Euclidean space \mathbf{C}^n or a complex hyperbolic space $H_n\mathbf{C}$ according as $c > 0$, $c = 0$ or $c < 0$.

In his study [12] of real hypersurfaces of $P_n\mathbf{C}$, Takagi showed that all homogeneous hypersurfaces could be divided into six types. Namely, he proved the following

THEOREM A. *Let M be a homogeneous real hypersurface of $P_n\mathbf{C}$. Then M is locally congruent to one of the following hypersurfaces:*

- (A₁) *a geodesic hypersurface,*
- (A₂) *a tube over a totally geodesic $P_k\mathbf{C}$ ($1 \leq k \leq n-2$),*
- (B) *a tube over a complex quadric Q_{n-1} ,*
- (C) *a tube over $P_1\mathbf{C} \times P_{(n-1)/2}\mathbf{C}$ and $n(\geq 5)$ is odd,*
- (D) *a tube over a complex Grassmann $G_{2,5}$ and $n=9$,*
- (E) *a tube over a Hermitian symmetric space $SO(10)/U(5)$ and $n=15$.*

Moreover, Takagi [13] proved that if a real hypersurface of $P_n\mathbf{C}$ has two or three distinct constant principal curvatures, then M is locally congruent to the case of the homogeneous ones of type A₁, A₂ or B. In what follows the induced almost contact metric structure of the real hypersurface of $M_n(c)$ is denoted by (ϕ, g, ξ, η) . The structure vector ξ is said to be *principal* if $A\xi = \alpha\xi$, where A is the shape operator in the direction of the unit normal C and $\alpha = \eta(A\xi)$. Real hypersurfaces of $P_n\mathbf{C}$ have been studied by many differential geometers ([2], [4], [5], [6] and [7] etc.) and as one of them, Kimura [5] asserts

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