

## A CHARACTERIZATION OF A REAL HYPERSURFACE OF TYPE B

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### Introduction.

A complex  $n$ -dimensional Kaehler manifold of constant holomorphic sectional curvature  $c$  is called a *complex space form*, which is denoted by  $M_n(c)$ . A complete and simply connected complex space form is a complex projective space  $P_n\mathbf{C}$ , a complex Euclidean space  $\mathbf{C}^n$  or a complex hyperbolic space  $H_n\mathbf{C}$  according as  $c > 0$ ,  $c = 0$  or  $c < 0$ .

In his study [12] of real hypersurfaces of  $P_n\mathbf{C}$ , Takagi showed that all homogeneous hypersurfaces could be divided into six types. Namely, he proved the following

**THEOREM A.** *Let  $M$  be a homogeneous real hypersurface of  $P_n\mathbf{C}$ . Then  $M$  is locally congruent to one of the following hypersurfaces:*

- (A<sub>1</sub>) a geodesic hypersurface,
- (A<sub>2</sub>) a tube over a totally geodesic  $P_k\mathbf{C}$  ( $1 \leq k \leq n-2$ ),
- (B) a tube over a complex quadric  $Q_{n-1}$ ,
- (C) a tube over  $P_1\mathbf{C} \times P_{(n-1)/2}\mathbf{C}$  and  $n(\geq 5)$  is odd,
- (D) a tube over a complex Grassmann  $G_{2,5}$  and  $n=9$ ,
- (E) a tube over a Hermitian symmetric space  $SO(10)/U(5)$  and  $n=15$ .

Moreover, Takagi [13] proved that if a real hypersurface of  $P_n\mathbf{C}$  has two or three distinct constant principal curvatures, then  $M$  is locally congruent to the case of the homogeneous ones of type A<sub>1</sub>, A<sub>2</sub> or B. In what follows the induced almost contact metric structure of the real hypersurface of  $M_n(c)$  is denoted by  $(\phi, g, \xi, \eta)$ . The structure vector  $\xi$  is said to be *principal* if  $A\xi = \alpha\xi$ , where  $A$  is the shape operator in the direction of the unit normal  $C$  and  $\alpha = \eta(A\xi)$ . Real hypersurfaces of  $P_n\mathbf{C}$  have been studied by many differential geometers ([2], [4], [5], [6] and [7] etc.) and as one of them, Kimura [5] asserts

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