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EXPANSIVENESS OF REAL FLOWS

Dedicated to Professor Yukihiro Kodama on his sixtieth birthday

By

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§1. Introduction.

Expansive transformations play important roles in topological dynamics. However there are several notions for expansiveness of real flows and the relationships between them have not been clarified enough. We investigate in this paper the relationship between some of expansive notions and show that the notions are unified into two kinds of expansiveness (Theorem and Theorem A). One is the expansiveness introduced by R. Bowen and P. Walters [2] and another is the weak expansiveness found in [5] in investigating the geometric Lorentz flow introduced by J. Guckenheimer [3].

Let X be a compact metric space with metric d and $\boldsymbol{R}$ denote the additive group of real numbers. A map $F: X\times\boldsymbol{R}\rightarrow X$ is called a *flow* on X if F is continuous and $f_{t+s}x=f_t(f_s x)$, $f_0 x=x$ for every $t, s\in\boldsymbol{R}$ and $x\in X$, where $f_t x=F(x, t)$.

R. Bowen and P. Walters introduced in [2] the notion of expansiveness as follows: A flow F is *expansive* if for any $\varepsilon>0$ there exists $\delta>0$ such that if $x, y\in X$ satisfy $d(f_t x, f_{s(s)}y)<\delta$ $(t\in\boldsymbol{R})$ for some continuous map $s; \boldsymbol{R}\rightarrow\boldsymbol{R}$ with $s(0)=0$, then $y=f_t x$ for some $|t|<\varepsilon$.

$f_I(S)=\{f_t x; t\in I, x\in S\}$ for an interval I and $S\subset X$. A flow F on X is called *weakly expansive* if F satisfies the property that for any $\varepsilon>0$ there exists $\delta>0$ with the property that if there exist a pair of points $x, y\in X$ and a strictly increasing surjective homeomorphism $h: \boldsymbol{R}\rightarrow\boldsymbol{R}$ with $h(0)=0$ such that $d(f_t x, f_{h(t)}y)<\delta$ for every $t\in\boldsymbol{R}$, then $f_{h(t_0)}y\in f_{(t_0-\varepsilon, t_0+\varepsilon)}(\{x\})$ for some $t_0\in\boldsymbol{R}$.

THEOREM (R. Bowen and P. Walters [2]). *The following are equivalent for a flow F.*

(i) *F is expansive.*

(ii) *For any $\varepsilon>0$ there exists $\delta>0$ such that if $x, y\in X$ satisfy $d(f_t x, f_{s(t)}y)$*

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