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KILLING VECTOR FIELDS AND THE HOLONOMY ALGEBRA IN SEMIRIEMANNIAN MANIFOLDS

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Abstract In this paper we generalize some results of Kostant [2] to semiriemannian manifolds of signature s. We also prove that any Killing vector field on a semiriemannian homogeneous compact flat manifold is parallel.

0. Introduction.

Let (M_s^n, g) be a semiriemannian manifold of dimension n and signature s. Let X be a Killing vector field on M. The A_X-operator provides a skew symmetric endomorphism of TM. It is well known that

$$\nabla_Y A_X = R_{XY}.$$

This fact and the Ambrossse-Singer theorem (Wo) show that the A_X-operator lies infinitesimally in the holonomy algebra $\boldsymbol{h}$ of M.

We ask ourselves whether or not A_X lies in $\boldsymbol{h}$.

In the riemannian case the question has an affirmative answer on compact manifolds [2]. We obtain here a similar result in the semiriemannian case.

Finally we study the holonomicity of a Killing vector field on semiriemannian manifolds of constant curvature. If the curvature is non zero, the holonomy algebra can be represented as $\boldsymbol{po}(n, s)$, that is the skew symmetric endomorphisms of TM. In this case each Killing vector field is holonomic.

There are flat manifolds and Killing vector fields on them such that the A_X-operator does not lie in the holonomy algebra, that is $A_X \notin \boldsymbol{h}$. Take, for instance, R_s^n. In the usual coordinates on R_s^n, X is a Killing vector field if

$$X = \sum_{i,j} \varepsilon_i K_i^j x_i \frac{\partial}{\partial x_j}$$

where $K_i^j = -K_j^i$ are constants, $\varepsilon_i = g(\partial/\partial x_i, \partial/\partial x_i) = \pm 1$ and $x_0 = 1$. There are nonholonomic Killing vector fields on R_s^n: nonparallel vector fields are nonholo-

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