TSUKUBA J. MATH.
Vol. 13 No. 2 (1989), 453—455

A NOTE ON MULTIVALENT FUNCTIONS

By

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The author would like to dedicate this paper to the memory of the late Professor Shigeo Ozaki.

It is well known that if $f(z)=z+\sum_{n=2}^{\infty}a_{n}z^{n}$ is analytic in $E=\{z:|z|<1\}$ and $Ref'(z)>0$ in E, then $f(z)$ is univalent in E.

Ozaki [3, Theorem 2] extended the above result to the following: if $f(z)$ is analytic in a convex domain D and

$$Re(e^{i\alpha}f^{(p)}(z))>0 \quad \text{in} \quad D$$

where α is a real constant, then $f(z)$ is at most p-valent in D.

This shows that if $f(z)=z^{p}+\sum_{n=p+1}^{\infty}a_{n}z^{n}$ is analytic in E and

$$Ref^{(p)}(z)>0 \quad \text{in} \quad E,$$

then $f(z)$ is p-valent in E.

DEFINITION. Let $F(z)$ be analytic and univalent in E and suppose that $F(E)=D$. If $f(z)$ is analytic in E, $f(0)=F(0)$, and $f(E)\subset D$, then we say that $f(z)$ is subordinate to $F(z)$ in E, and we write

$$f(z)\prec F(z).$$

In this paper, we need the following lemmata.

LEMMA 1. *If* $p(z)$ *is analytic in* E, *with* $p(0)=1$, *and*

$$p(z)+zp'(z)\prec\left(\frac{1+z}{1-z}\right)^{3/2} \quad in \quad E,$$

then we have

$$p(z)\prec\frac{1+z}{1-z} \quad in \quad E.$$

We owe this lemma to [1, Theorem 5 and its remark].

REMARK. From Lemma 1, it is trivial that if $p(0)=1$ and

Received August 24, 1988.