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HYPOELLIPTICITY OF SYSTEMS OF PSEUDO DIFFERENTIAL OPERATORS WITH DOUBLE CHARACTERISTICS

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0. Introduction.

In this paper, we are concerned with the hypoellipticity of a system of pseudodifferential operators on Ω, an open subset of R^N, of the form

$$P(x, D)=I_d p(x, D)+Q(x, D), \tag{0.1}$$

where I_d is the identity $d\times d$ matrix, $p(x, D)$ a scalar pseudodifferential operator of degree m, and $Q(x, D)$ a $d\times d$ system of pseudodifferential operators of degree at most $m-1$. We shall assume that its principal symbol $p(x, \xi)$ is nonnegative on $T^*\Omega$, the cotangent bundle of Ω and that it vanishes exactly of order 2 on its characteristic set Σ, which is assumed to be a symplectic smooth submanifold of $T^*\Omega$. In [3], L. Boutet de Monvel and F. Treves have obtained a necessary and sufficient condition for $P(x, D)$ such as above (in fact, a little more general ones) to be hypoelliptic with loss of one derivative, which is the best hypoellipticity for $P(x, D)$ to have. In case of a scalar situation (i.e. $d=1$), hypoellipticity (and local solvability) of $P(x, D)$ was studied in [6], assuming, in addition, that the codimension of Σ in $T^*\Omega$ is 2, in which case the analysis is much simpler than the present case. In this work, we obtain sufficient conditions for $P(x, D)$ to be hypoelliptic, which extend the results in [6] to the vector situation with no restriction on codimension of Σ. As in [6], we rely heavily on the method of concatenations which was initiated by F. Treves in [8] and turned out to be quite useful in some cases (cf. [2, 3, 5, 6, 8]). After reducing the operator under study into a canonical form near a characteristic point in section 1, we construct, in section 2, a series of operators, called concatenations, by which we can reformulate the condition under which $P(x, D)$ is hypoelliptic with loss of one derivative. Then, in section 3, we state and prove the main results of this paper.

We use $(x, \xi)=(x_1, \cdots, x_N, \xi^1, \cdots, \xi^N)$ for the variable point in $T^*\Omega$ and

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