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REFLEXIVE MODULES AND RINGS WITH SELF-INJECTIVE DIMENSION TWO

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Let R be a left and right noetherian ring and M a finitely generated left R-module with $\mathrm{Ext}_R^i(M, R)=0$ for $i\geqq 1$. Is then M reflexive ? This is a stronger version of the generalized Nakayama conjecture posed by Auslander and Reiten [2]. In this note, we ask when every finitely generated left R-module M with $\mathrm{Ext}_R^i(M, R)=0$ for $i=1, 2$ is reflexive. Our main aim is to show that if R is a left and right noetherian ring then $\mathrm{inj\,dim}_R R=\mathrm{inj\,dim}\, R_R\leqq 2$ if and only if for a finitely generated left R-module M the following conditions are equivalent: (1) M is reflexive; (2) there is an exact sequence $0\rightarrow M\rightarrow P_1\rightarrow P_0$ of left R-modules with the P_i projective; and (3) $\mathrm{Ext}_R^i(M, R)=0$ for $i=1, 2$. We will show also that if R is a commutative noetherian ring then it is a Gorenstein ring of dimension at most two if and only if the ring of total quotients of R is a Gorenstein ring and every finitely generated R-module M with $\mathrm{Ext}_R^i(M, R)=0$ for $i=1, 2$ is reflexive.

In what follows, R stands for a ring with identity, and all modules are unital R-modules. We denote by $(\ )^*$ both the R-dual functors, and for a module M we denote by $\varepsilon_M: M\rightarrow M^{**}$ the usual evaluation map. Recall that a module M is said to be torsionless if ε_M is a monomorphism and to be reflexive if ε_M is an isomorphism. Also, a module M is said to be finitely presented if it admits an exact sequence $P_1\rightarrow P_0\rightarrow M\rightarrow 0$ with the P_i finitely generated and projective. Note that if R is left noetherian then every finitely generated left module is finitely presented.

1. Preliminaries

In this section, we prepare several lemmas which we need in the next section.

LEMMA 1.1. *The following are equivalent*:

(1) *Every finitely presented left module M with* $\mathrm{Ext}_R^i(M, R)=0$ *for* $i=1, 2$

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