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ALMOST-PRIMES IN ARITHMETIC PROGRESSIONS AND SHORT INTERVALS

By

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1. Introduction.

In 1936, P. Turán [9] showed, under the generalized Riemann hypothesis, there exists a prime p such that

$$p\equiv a \pmod{q}, \quad p\leqq q (\log q)^{2+\varepsilon}$$

for almost-all reduced classes a $(\bmod\, q)$. The terminology almost-all means that the number of exceptional reduced classes $\bmod\, q$ is $o(\phi(q))$ as $q\to\infty$. Y. Motohashi [6] considered the corresponding problem for almost-primes. Let P_2 denote integers with at most two prime factors, multiple factors being counted multiplicity. He proved that there exists a P_2 such that

$$P_2\equiv a \pmod{q}, \quad P_2\leqq q^{11/10}$$

for almost-all reduced classes a $\bmod\, q$. Moreover he remarked, assuming the q-analogue of Lindelöf hypothesis, the exponent 11/10 may be replaced by $1+\varepsilon$, $\varepsilon>0$.

It is the first purpose of this paper to make an improvement upon this result. Let $g(x)$ denote any positive function such that $g(x)\to\infty$ as $x\to\infty$.

THEOREM 1. *There exists a P_2 such that*

$$P_2\equiv a \pmod{q}, \quad P_2\leqq g(q)q(\log q)^5$$

for almost-all reduced classes a $\bmod\, q$.

In 1943, A. Selberg [8] showed, under the Riemann hypothesis, there exists a prime in the intervals

$$(n, n+g(n)(\log n)^2]$$

for almost-all n. Here almost-all means that the number of exceptional n's not exceeding x is $o(x)$ as $x\to\infty$.

Several authors considered the analogous problem for P_2. Thus D. R. Heath-

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