TSUKUBA J. MATH.
Vol. 13 No. 2 (1989), 363—386

ON THE CAUCHY PROBLEM FOR QUASILINEAR HYPERBOLIC-PARABOLIC COUPLED SYSTEMS IN HIGHER DIMENSIONAL SPACES

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0. Introduction

In this paper, we consider the Cauchy problem for quasilinear hyperbolic-parabolic coupled systems of second order.

$$\partial_t^2\vec{u}-\partial_i A^i(t, x, \theta+T_0, \nabla\vec{u})=\vec{f}(t, x, \theta+T_0, \vec{u}, \nabla\vec{u}, \partial_t\vec{u}), \tag{0.1}$$

$$(\theta+T_0)\partial_t N(t, x, \theta+T_0, \nabla\vec{u})-\partial_i Q^i(t, x, \nabla\theta, \nabla\vec{u}) =g(t, x, \theta+T_0, \nabla\theta, \vec{u}, \nabla\vec{u}, \partial_t\vec{u}), \tag{0.2}$$

$$\vec{u}(0, x)=\vec{u}_0(x), \partial_t\vec{u}(0, x)=\vec{u}_1(x), \theta(0, x)=\theta_0(x), \tag{0.3}$$

where $x\in \boldsymbol{R}^m$ and $t\in[0, T]$. Here and hereafter $\vec{v}={}^t(v_1, \cdots, v_n)$ (${}^tM$ means the transpose M); $\vec{u}$ and θ are unknown functions; T_0 is a positive constant. $\partial_i=\partial/\partial x_i$ and $\partial_t=\partial/\partial t$; ∇ denotes the gradient in x, the summation convention is understood such as sub and superscripts i and j take all values 1 to m; $A^i={}^t(A_1^i, \cdots, A_n^i)$, $\vec{f}={}^t(f_1, \cdots, f_n)$, N, Q^i and g are given nonlinear functions.

(0.1) and (0.2) can be easily extended to the certain kind of hyperbolic and parabolic equations respectively and arise from the thermoelastodynamics theory (cf [9]). On this kind of coupled systems with hyperbolic systems of second order and parabolic systems of second order, Slemrod [14] studied in the 1-dimensional case. We will show the local and unique solvability of (0.1), (0.2) and (0.3) with certain assumptions.

Our approach is the following: apply the existence theorems of solutions to linear coupled systems of 2nd order hyperbolic system and parabolic equations and estimations obtained by the energy method. Another approach is the following; reduce (0.1), (0.2) and (0.3) to the coupled systems of 1st order hyperbolic systems and 2nd order parabolic equations, and apply the theory due to Kawashima [8] or Zheng [17]. But it seems that our approach is the best regarding the minimal order of Sobolev spaces in which solutions exist. Another advantage of our approach is that we can handle the complicated nonlinear

Received May 25, 1988.