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CORINGS AND INVERTIBLE BIMODULES

By

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Introduction.

Let $S \subset R$ be a faithfully flat extension of commutative rings (with 1). Grothendieck's faithfully flat descent theory tells that the relative Picard group $\mathrm{Pic}(R/S)$ is isomorphic to $H^1(R/S, U)$, the Amitsur 1-cohomology group for the units-functor U. We consider the non-commutative version of this fact in this paper.

Let $S \subset R$ be (non-commutative) rings and denote by $\mathrm{Inv}_S(R)$ the group of invertible S-subbimodules of R. Sweedler defined the natural R-coring structure on $R \otimes_S R$. We define the natural group map $\boldsymbol{\Gamma}: \mathrm{Inv}_S(R) \to \mathrm{Aut}_{R\text{-cor}}(R \otimes_S R)$, where $\mathrm{Aut}_{R\text{-cor}}(R \otimes_S R)$ denotes the group of R-coring automorphisms of $R \otimes_S R$. When is $\boldsymbol{\Gamma}$ an isomorphism ? The answer presented here is as follows (2.10): *If either*

(a) *R is faithfully flat as a right or left S-module*

or (b) *S is a direct summand of R as a right* (resp. *left*) *S-module and the functor* $-\otimes_S R$ (resp. $R \otimes_S -$) *reflects isomorphisms,*

then $\boldsymbol{\Gamma}$ is an isomorphism. Indeed we consider some monoid map $\mathbf{I}^l_S(R) \to \mathrm{End}_{R\text{-cor}}(R \otimes_S R)$, which is an extension of $\boldsymbol{\Gamma}$. We have two applications (3.2) and (3.4), both of which are concerned with the Galois theory.

§ 0. Conventions.

Let T, Q be arbitrary rings with 1. We write

$$U(T) = \text{the group of units in } T.$$

All modules are assumed to be unital. A (T, Q)-bimodule means a left T-module and right Q-module M satisfying $(tm)q = t(mq)$ for $t \in T$, $m \in M$ and $q \in Q$. A T-bimodule means a (T, T)-bimodule. We denote by

$${}_T\mathcal{M}, \quad \mathcal{M}_T \quad \text{and} \quad {}_T\mathcal{M}_Q$$

the category of left T-modules, of right T-modules and of (T, Q)-bimodules,

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