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EQUIVARIANT CW COMPLEXES AND SHAPE THEORY

Dedicated to Professor Masahiro Sugawara on his 60th birthday

By

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The aim of this note is to study a discrete group equivariant shape theory by associating an inverse system in the homotopy category of equivariant CW complexes.

1. Introduction

Let G be a discrete group and X a G-space. For a subgroup H of G we denote $X^{H}=\{x\in X; gx=x$ for every $g\in H\}$. For a G-map $f: X\rightarrow Y$ of X to another G-space Y, we denote $f^{H}=f|X^{H}: X^{H}\rightarrow Y^{H}$. Let $\mathcal{H}_{G}$ denote the category of G-spaces and G-homotopy classes of G-maps and $\mathcal{W}_{G}$ the full subcategory of $\mathcal{H}_{G}$ consisting of G-spaces which have the G-homotopy types of G-CW complexes.

THEOREM 1. *There is a functor* $\check{C}_{G}$ *from* $\mathcal{H}_{G}$ *into the pro-category* pro-$\mathcal{W}_{G}$ *of* $\mathcal{W}_{G}$ *so that* $\check{C}_{G}(X)=(X_{\lambda}, [p_{\lambda\lambda'}^{X}]_{G}, \Lambda)$ *has the universal property for the equivariant shape theory with a system G-map* $p^{X}=([p_{\lambda}^{X}]_{G}): X\rightarrow\check{G}_{G}(X)$, *that is,* $p^{X}: X\rightarrow\check{C}_{G}(X)$ *is a G*-CW *expansion of X.*

When G is a finite group, we know that a G-ANR has the G-homotopy type of a G-CW complex and vice versa. Also any numerable covering has a refinement of numerable G-equivariant covering. So, we have

TNEOREM 2. *Let G be a finite group and X a G-space.*

(1) *Any G*-ANR *expansion of X is equivalent to* $p^{X}: X\rightarrow\check{C}_{G}(X)$.

(2) *The expansion* $p^{X}: X\rightarrow\check{C}_{G}(X)$ *is a (non-equivariant)* CW *expansion of X. Moreover, if X is a normal G-space, then* $p^{X,H}=([p_{\lambda}^{X,H}]): X^{H}\rightarrow\check{C}_{G}(X)^{H}=(X_{\lambda}^{H}, [p_{\lambda\lambda'}^{X,H}], \Lambda)$ *is a* CW *expansion for every subgroup H of G.*

(3) *Let* $f: X\rightarrow Y$ *be a G-map between normal G-spaces. Then,* $\check{C}_{G}(f): \check{G}_{G}(X)\rightarrow\check{C}_{G}(Y)$ *is an isomorphism in* pro-$\mathcal{W}_{G}$ *if and only if* $f^{H}: X^{H}\rightarrow Y^{H}$ *is a shape*

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