TSUKUBA J. MATH.
Vol. 13 No. 1 (1989). 99–105

A GENERALIZATION OF A RESULT OF K. R. JOHNSON

By

Don REDMOND

In [2], Grytczuk showed that

$$\sum_{d \mid k} |c_d(n)| = z^{\omega(k/(k,n))}(k, n),$$

where $c_k(n)$ denotes Ramanujan's trigonometric sum and $\omega(m)$ counts the number of distinct prime divisors of m. In [3], Johnson evaluated the sum

$$\sum_{d \mid n} |c_k(d)| .$$

In [4], I generalized the result of Grytczuk to a larger class of functions. In this paper I generalize the result of Johnson.

If h is an arithmetic function, we define the arithmetic function H_k by

$$H_k(n) = \sum_{d \mid (k,n)} \mu(k/d) h(d). \tag{1}$$

In [4] it is shown that $H_1(n) = h(1)$, if $a \geqq 1$,

$$H_{p^a}(n) = \begin{cases} h(p^a) - h(p^{a-1}) & \text{if } p^a \mid n \\ -h(p^{a-1}) & \text{if } p^{a-1} \parallel n \\ 0 & \text{if } p^{a-1} \nmid n \end{cases} \tag{2}$$

and that $H_k(n)$ is a multiplicative function of k. In [4], we investigated the sum

$$\sum_{d \mid k} |H_d(n)| .$$

and in this paper we shall investigate the sum

$$\sum_{d \mid n} |H_k(d)| .$$

Since $H_k(n)$ is not a multiplicative function of n, this task is a little more difficult. We shall assume throughout this paper that h is a multiplicative function.

To make our generalization of Johnson's result as clear as possible we shall follow his notation as closely as possible. In particular, for a given positive integer k we denote by $\bar{k}$ the core of k, that is, the largest square-free divisor of k, and we denote by k^* the integer $k/\bar{k}$.

Received March 11, 1988.