

THE SHRINKING PROPERTY OF Σ -PRODUCTS

By

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1. Introduction.

Concerning the study of the normality of Σ -products, the following results have been proved in order:

(A) A Σ -product of metric spaces is normal (by Gul'ko [6] and Rudin [15] in 1977).

(B) A Σ -product of paracompact p -spaces is normal iff it has countable tightness (by Kombarov [8] in 1978).

(C) A Σ -product of paracompact Σ -spaces is normal if it has countable tightness (by the author [17] in 1984).

On the other hand, the shrinking property is between paracompactness and normality. Rudin [16] in 1983 began to study the shrinking property of Σ -products and LeDonne [10] in 1985 extended her results. That is, they respectively proved the following:

(A') A Σ -product of metric spaces is shrinking.

(B') A Σ -product of paracompact p -spaces is shrinking iff it is normal.

The main purpose of the present paper is to prove the further extension, according to (C), as follows:

(C') A Σ -product of strong Σ -spaces is shrinking iff it is normal. Moreover, we prove that the "strong Σ -spaces" in (C') can be replaced by "semi-metric spaces". This gives another generalization of (A').

The weak \mathcal{B} -property is weaker than the shrinking one. Chiba [2] proved that a Σ -product of compact spaces has the weak \mathcal{B} -property. So she asked in [3] whether a Σ -product of paracompact M -spaces ($=p$ -spaces) has the weak \mathcal{B} -property. Here, we give an affirmative answer to this question.

All results proved here were early announced in [19] as a report.

All spaces are assumed to be regular T_1 . The letters n, m, k, i, j and l denote non-negative integers.