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UNIVERSAL TRANSITIVITY OF CERTAIN CLASSES OF REDUCTIVE PREHOMOGENEOUS VECTOR SPACES

By

Shin-ichi KASAI

Introduction.

Let k be a field of characteristic zero. Let $\tilde{G}$ be a connected k-split linear algebraic group, $\rho: \tilde{G}\rightarrow GL(X)$ with $X=\mathrm{Aff}^{n}$ a k-homomorphism. If there exists a Zariski-dense $\rho(\tilde{G})$-orbit Y, we say that $(\tilde{G}, \rho, X)$ is a *prehomogeneous vector space* (abbrev. *P. V.*). When each irreducible component is castling equivalent to a non-trivial reduced irreducible prehomogeneous vector space or each irreducible component is a regular prehomogeneous vector space, we have completed a classification of reductive prehomogeneous vector spaces over a complex number field $\mathbf{C}$ (see [4], [5]).

Put $G=\rho(\tilde{G})$. Let l be the number of $G(k)$-orbits in $Y(k)$, i.e., $l=|G(k)\backslash Y(k)|$. We say that Y is a *universally transitive open orbit* if $l=|G(k)\backslash Y(k)|=1$ for all k satisfying $H^{1}(k, \mathrm{Aut}(SL_{2}))\neq 0$, i.e., there exists a nonsplit quaternion k-algebra. This condition is satisfied by every local field k other than $\mathbf{C}$. Actually our classification depends on the transitivity of $G(k)$ on $Y(k)$ for *just one* k satisfying $H^{1}(k, \mathrm{Aut}(SL_{2}))\neq 0$ (see Remarks 2.13 and 3.5). In [1] and [2], all irreducible regular prehomogeneous vector spaces with universally transitive open orbits are classified. In [10], we have classified simple or 2-simple prehomogeneous vector spaces with universally transitive open orbits.

In this paper, we shall classify reductive prehomogeneous vector spaces with universally transitive open orbits when each irreducible component is castling equivalent to a non-trivial reduced irreducible prehomogeneous vector space or each irreducible component is a regular prehomogeneous vector space.

This paper consists of the following three sections.

§1. Preliminaries.

§2. Reductive P.V.'s with universally transitive open orbits: the case I.

§3. Reductive P.V.'s with universally transitive open orbits: the case II.

The results are given in Theorems 2.11, 3.4 and Corollary 2.12.

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