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A DIRECT PROOF THAT EACH PEANO CONTINUUM WITH A FREE ARC ADMITS NO EXPANSIVE HOMEOMORPHISMS

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A homeomorphism $f: X\rightarrow X$ of a compact metric space X is said to be *expansive* if there exists a constant $c>0$ (called *expansive constant*) such that

(*) for each pair x, y of distinct points of X, there exists an integer n such that $d(f^{n}(x), f^{n}(y))>c$, where d is a metric for X. Expansiveness does not depend on the choice of metrics for compact metric spaces.

A compact connected metric space is called a *continuum*. A *Peano continuum* means a locally connected continuum. An arc A in a continuum X with end points $\{a, b\}$ is denoted by $[a, b]$. $bd\,A$ means $\{a, b\}$ and $\mathrm{int}\,A=A-bd\,A$. An arc A in X is called a *free arc* if $\mathrm{int}\,A$ is open in X. Let (X, d) be a continuum. For a point $x\in X$ and $\varepsilon>0$, $U(x, \varepsilon)$ denotes the ε-neighbourhood of x. The Hausdorff metric is denoted by d_{H}.

In this paper, we give a direct proof of the following theorem, which is a consequence of Proposition C in Hiraide [2].

THEOREM. *Let X be a Peano continuum with a free arc. Then there does not exist expansive homeomorphisms of X.*

The author benefits from reading Proposition C in [2] and wishes to thank to Professor K. Sakai for his helpful suggestions.

First we list known results which are necessary for the proof of Theorem.

LEMMA 1 ([3] p. 257, theorem 4). *Let (X, d) be a Peano continuum. For each $\varepsilon>0$, there exists a $\delta>0$ such that each pair of points $x, y\in X$ with $d(x, y)<\delta$ can be joined by an arc whose diameter is less than ε.*

LEMMA 2 ([3] p. 179, theorem 1). *A continuum X is homeomorphic to an arc if and only if there exist two points a and b of X such that*

1) *$X-a$ and $X-b$ are connected and*
2) *for each $x\in X$ with $a\neq x\neq b$, $X-x$ is not connected.*

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