TSUKUBA J. MATH.
Vol. 12 No. 2 (1988) 497—505

ISOTROPIC MINIMAL SUBMANIFOLDS IN A SPACE FORM

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Let $\tilde{M}^m(\tilde{c})$ be an *m-dimensional space form* of constant curvature $\tilde{c}$, that is, an m-dimensional Riemannian manifold of constant curvature $\tilde{c}$. By the Theorem in [5], the author determined n-dimensional minimal submanifolds in $\tilde{M}^m(\tilde{c})$ with the sectional curvature not less than $n\tilde{c}/2(n+1)$. We should pay attention to the value next to $n\tilde{c}/2(n+1)$, so that we could classify minimal submanifolds in $\tilde{M}^m(\tilde{c})$ with the sectional curvature not less than it.

In the present paper, we will classify n-dimensional isotropic minimal submanifolds in $\tilde{M}^m(\tilde{c})$ with the sectional curvature not less than some value. Indeed, we will prove the following

THEOREM A. *Let M be a connected compact n-dimensional $(n\geqq 3)$ orientable submanifold isotropically and minimally immersed in an $(n+\nu)$-dimensional space form $\tilde{M}$ of constant curvature $\tilde{c}$. If the sectional curvature of M is not less than $n\tilde{c}/3(n+2)$, then M is of constant curvature $\tilde{c}$ or $n\tilde{c}/3(n+2)$, or the second fundamental form is parallel.*

We may assume that $0<\tilde{c}$ by (2.17) and Remark in §2, that is, $\tilde{M}$ is a sphere $S^m(\tilde{c})$ of constant curvature $\tilde{c}$. When M is of constant curvature, by the results in [2], according as the sectional curvature is $\tilde{c}$ or $n\tilde{c}/3(n+2)$, M is a great sphere of $S^m(\tilde{c})$ or the immersion is *the standard minimal one of degree* 3 *from a shere into a sphere* as stated in [2], which we will call *the generalized Veronese submaniolfd* in the present paper. When the second fundamental form is parallel, the above immersion is the planar geodesic one, which is determined in [8]. As a Corollary to Theorem A, using the results in [2], [4] and [8], we have the following

THEOREM B. *Let M be a connected compact n-dimensional $(n\geqq 3)$ orientable submanifold minimally and isotropically immersed in a sphere $S(\tilde{c})$ of constant curvature $\tilde{c}$. If the immersion is full and the sectional curvature K_σ satisfies the inequality: $n\tilde{c}/3(n+2)\leqq K_\sigma\leqq\tilde{c}$, then M is a great sphere of $S(\tilde{c})$, a Veronese submanifold, or a generalized Veronese submanifold.*

Received October 22, 1987, Revised December 22, 1987.