TSUKUBA J. MATH.
Vol. 12 No. 2 (1988), 489–495

HOMEOMORPHISMS OF ZERO-DIMENSIONAL SPACES

By

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Dedicated to Professor Yukihiro Kodama on his 60th birthday

1. Introduction.

All spaces considered in this paper are assumed to be compact and metrizable.

Let φ be a homeomorphism from a space (X, d) onto itself. Then φ is *expansive* if there is $c>0$ such that for every $x, y\in X$ with $x\neq y$ there is $n\in \boldsymbol{Z}$ for which $d(\varphi^{n}(x), \varphi^{n}(y))>c$. Given $\delta>0$, a sequence $\{x_{i}: i\in \boldsymbol{Z}\}$ is a *δ-pseudo-orbit* of φ if $d(\varphi(x_{i}), x_{i+1})<\delta$ for every $i\in \boldsymbol{Z}$. Given $\varepsilon>0$, a sequence $\{x_{i}: i\in \boldsymbol{Z}\}$ is *ε-traced* by a point $y\in X$ if $d(\varphi^{i}(y), x_{i})<\varepsilon$ for every $i\in \boldsymbol{Z}$. We say that φ has the *pseudo orbit tracing property* (abbrev. P.O.T.P.) if for every $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of φ can be ε-traced by some point of X.

For a space (X, d) we denote by $\mathcal{H}(X)$ the space of all homeomorphisms of X with the metric $\bar{d}(\varphi, \psi)=\max\{d(\varphi(x), \psi(x)): x\in X\}$ for every $\varphi, \psi\in \mathcal{H}(X)$. Let $\mathcal{E}(X)=\{\varphi\in \mathcal{H}(X): \varphi$ is expansive$\}$ and $\mathcal{P}(X)=\{\varphi\in \mathcal{H}(X): \varphi$ has P.O.T.P.$\}$.

In Section 3 we are concerned with the Cantor set C. The Cantor set C is the unique zero-dimensional infinite group. N. Aoki [1] proved that every group automorphism of C has P.O.T.P. M. Sears [6] proved that $\mathcal{E}(C)$ is dense in $\mathcal{H}(C)$, constructing a dense subset $\mathcal{A}$ of $\mathcal{E}(C)$ in $\mathcal{H}(C)$. M. Dateyama [3] proved that $\mathcal{P}(C)$ is dense in $\mathcal{H}(C)$, constructing a dense subset $\mathcal{B}$ of $\mathcal{P}(C)$ in $\mathcal{H}(C)$. However, for the sets $\mathcal{A}$ and $\mathcal{B}$ above we have $\mathcal{A}\cap\mathcal{B}=\phi$. So it is unknown whether the set $\mathcal{E}(C)\cap\mathcal{P}(C)$ of all expansive homeomorphisms with P.O.T.P. of C is dense in $\mathcal{H}(C)$. In Section 3 we shall prove the following theorem.

THEOREM 1. *The set of all expansive homeomorphisms with* P.O.T.P. *of the Cantor set C is dense in $\mathcal{H}(C)$.*

We know [6] that $\mathcal{E}(C)$ is of first category. So $\mathcal{E}(C)\cap\mathcal{P}(C)$ is also of first category.

The convergent sequence is another standard zero-dimensional space, classed with the Cantor set. In Section 4 we shall prove the following theorem.

Received October 22, 1987