TSUKUBA J. MATH.
Vol. 12 No. 2 (1988). 477-487

REMARKS ON TRANSMISSION, ANTITRANSMISSION AND ANTILOCAL PROPERTIES FOR SUMS OF STABLE GENERATORS

By

Yasushi ISHIKAWA

§1. Introduction.

The antilocal property (to all directions) of an operator A states that, if $f=Af=0$ in a domain U, then f is identically zero on the whole space. (For the precise definition see §2). This property is known to hold for the class of fractional powers of the Laplacian: $A=\Delta^{\lambda}$, where λ is a non-integral real number, by Goodman-Segal [**6**] when the space dimension is odd and by Murata [**14**] in general. Liess [**13**] showed this property for the fractional powers of the elliptic differential operator with analytic coefficients using the theory of pseudodifferential operators with analytic symbol.

As is well known, the fractional power $\Delta^{\alpha/2}$ is the generator of the isotropic stable process in case $0<\alpha<2$. In this case the antilocal property has an interesting application for the uniqueness problem of measures of Riesz potentials. See Kanda [**10**].

The class of generators for α-stable processes (called the stable generators) includes a class of operators which are different from $\Delta^{\alpha/2}$, for example, that of one-sided α-stable generators. For details see §4. The author [**9**] showed that if A is a one-dimensional one-sided α-stable generator (to the right), A has a biased antilocal property in the sense: if $f=Af=0$ on $(0, \varepsilon)$, $f\in C_0^{\infty}(\boldsymbol{R})$, $\varepsilon>0$, then $f\equiv 0$ on $(0, +\infty)$ but not necessarily $f\equiv 0$ in $(-\infty, 0)$. The result has been shown for $\alpha\in(0, 1)$ and will be extended to $\alpha\in(0, 1)\cup(1, 2)$ in §4, b), 2). [It is still open if the asymmetric Cauchy generators (the generalized case for $\alpha=1$) have the similar property.] The proof is carried out on the similar line as in Liess [**13**], and so Sato-Kashiwara-Kawai theory is essential in the proof.

The first aim of this note is to show what type of antilocal property holds for a finite sum of one-dimensional stable generators, for example, the sum of a one-sided α-stable generator to the right and a one-sided β-stable generator to the left. The solution will be given in §4, b), 3). In fact the example cited

Received October 20, 1987.