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ON HELICES AND PSEUDO-RIEMANNIAN SUBMANIFOLDS

By

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§ 0. Introduction.

In a Riemannian manifold, a regular curve is called a helix if its first and second curvature is constant and the third curvature is zero. As for helices in a Riemannian submanifold, there is a research of T. Ikawa, who investigated the condition that every helix with curvatures k, l in a Riemannian submanifold is a helix in the ambient space [**3**]. In a pseudo-Riemannian manifold, helices are defined by almost the same way as the Riemannian case. Recently, T. Ikawa proved the following theorem about helices in a Lorentzian submanifold [**4**]:

THEOREM A. *Let* M_1 ($\dim M_1 \geqq 3$) *be a Lorentzian submanifold of a pseudo-Riemannian manifold* $\tilde{M}_\beta$. *For any positive constant* k, l, *the following conditions are equivalent:*

(a) *every helix in* M_1 *with* $\langle X, X\rangle=-1$, $\langle \nabla_X X, \nabla_X X\rangle=k^2$ *and* $\langle \nabla_X\nabla_X X, \nabla_X\nabla_X X\rangle =-k^4+k^2l^2$ *is a helix in* $\tilde{M}_\beta$,

(b) M_1 *is totally geodesic.*

In this paper, we generalize this theorem to the case of a pseudo-Riemannian submanifold.

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§ 1. Preliminaries.

Let V_α be an n-dimensional real vector space equipped with an inner product $\langle , \rangle$ of index α. A non-zero vector x of V_α is said to be *null* if $\langle x, x\rangle=0$ and *unit* if $\langle x, x\rangle=+1$ or -1. Concerning multilinear mappings on V_α, we have the following lemmas [1]:

LEMMA 1.1. *For any r-linear mapping* T *on* V_α *to a real vector space* W *and* $\varepsilon_0=+1$ *or* -1 ($-\alpha \leqq \varepsilon_0 \leqq n-\alpha$), *the following conditions are equivalent:*

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