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A CHARACTERIZATION OF GORENSTEIN ORDERS

Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

By

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Introduction.

In this paper, we give a characterization of Gorenstein orders over a commutative d-dimensional Gorenstein local ring (Theorem 1.1) and study some special classes of Gorenstein orders (Theorem 2.1). Auslander called an order Λ Gorenstein if $\Lambda^* \cong \Lambda$ as Λ-Λ-bimodules [2]. Our definition of Gorenstein orders is more general and there are other interesting orders between our sense of Gorenstein orders and that of Auslander's. These orders are studied in Theorem 2.1.

Let R be a commutative d-dimensional Gorenstein local ring with its maximal ideal $\mathbf{m}$. Following Auslander [2], an R-algebra Λ is called an *R-order* if Λ is a finitely generated maximal Cohen-Macaulay R-module such that $\mathrm{Hom}_R(\Lambda, R)_{\mathbf{p}}$ is a projective $\Lambda_{\mathbf{p}}^{op}$-module for all nonmaximal prime ideals $\mathbf{p}$ of R. We call an R-order Λ *Gorenstein* if $\Lambda^* = \mathrm{Hom}_R(\Lambda, R)$ is a projective Λ^{op}-module. It is easily seen that the definition of Gorenstein orders is left-right symmetric and also that Λ^* is a progenerator for mod Λ when Λ is Gorenstein. In a classical case, that is, R being a discrete valuation ring, Gorenstein orders and their representation theories are widely studied (cf. [4, 7]). They are mostly the parallel results with those of QF algebras over a field. Gorenstein orders include both classical Gorenstein orders and QF algebras over a field. Thus many results for classical and algebra cases can be extended to our cases, and we give the most basic ones in sections 1, 2. In section 3, we give various examples concerning Gorenstein orders.

We define some more definitions and notation. We call M a Λ-lattice if it is a finitely generated Λ-module and a maximal Cohen-Macaulay R-module such that, $M_{\mathbf{p}}$, respectively $\mathrm{Hom}_R(M, R)_{\mathbf{p}}$ is a projective $\Lambda_{\mathbf{p}}$, respectively $\Lambda_{\mathbf{p}}^{op}$-module for all nonmaximal prime ideals $\mathbf{p}$ of R. The category of all Λ-lattices is denoted by $\mathcal{L}(\Lambda)$. All modules are considered as right modules. Left modules

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