TSUKUBA J. MATH.
Vol. 12 No. 2 (1988). 441—449

TILTING MODULES, DOMINANT DIMENSION AND EXACTNESS OF DUALITY FUNCTORS

By

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Dedicated to Professor Hiroyuki Tachikawa for his sixtieth birthday

Let R and S be rings and let ${}_RW_S$ be a bimodule. We shall denote both the functors $\mathrm{Hom}_R(\_, W)$: R-Mod$\rightarrow$Mod-S and $\mathrm{Hom}_S(\_, W)$: Mod-$S\rightarrow R$-Mod by Δ_W and the composition of the two, in either order, by Δ_W^2. Recall that (for fixed W) there is a natural transformation $\delta: 1_{R\text{-Mod}}\rightarrow\Delta_W^2$, defined via the usual evaluation maps $\delta_M: M\rightarrow\Delta_W^2(M)$. An R-module M is called *W-reflexive* (*W-torsionless*) in case δ_M is an isomorphism (a monomorphism). Then, an R-module M is W-torsionless if and only if it is isomorphic to a submodule of a direct product of copies of ${}_RW$. Also recall that ${}_RW_S$ is *balanced* in case $R\cong\mathrm{End}_S(W)$ and $S\cong\mathrm{End}_R(W)^{op}$ canonically, and that ${}_RW_S$ defines a *Morita Duality* if it is balanced and both ${}_RW$ and W_S are injective cogenerators (see [1], [3] or [10], for an account of Morita Duality).

We begin by studying exactness properties of the functor Δ_W^2. The case $W=R$ has been extensively studied in ([4], [5], [6] and [7]) and Theorem 1, Lemma 2, Proposition 3 and Proposition 4 are generalizations of results obtained there. A finite dimensional algebra R of positive dominant dimension possesses (what we consider to be) a canonical pair of tilting left and right modules ${}_RU$ and V_R. Associated with these are the endomorphism rings $S=\mathrm{End}_R(U)^{op}$ and $T=\mathrm{End}_R(V)$, and the bimodule ${}_TW_S={}_T(V\otimes_RU)_S$. We relate exactness properties of the functors Δ_U, Δ_V, Δ_W and their squares to dominant dimension. For these canonically chosen tilting modules we show that

1) if dom. dim. $R\geqq 2$ then Δ_U^2 preserves monomorphisms both in Mod-S and in R-Mod;

2) if dom. dim. $R\geqq 3$ then Δ_U^2 is left exact on Mod-S and the functors Δ_W^2 preserve monomorphisms in Mod-S and in T-Mod. In this case, if Δ_W: T-Mod $\leftrightarrow$Mod-S: Δ_W defines a Morita Duality, then R is QF (and conversely);

3) if dom. dim. $R\geqq 4$ then the functors Δ_W^2 are left exact on both Mod-S and on T-Mod.

We shall denote the injective envelope of a module M by $E(M)$ and, if M is an R-module, we denote the annihilator in M of a subset I of R by $\mathrm{Ann}_M(I)$.

Received September 1, 1987.