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AN INTERPRETATION OF INTUITIONISTIC ANALYSIS WITH RESTRICTED TRANSFINITE INDUCTIVE DEFINITIONS

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Introduction.

In some exact sciences such as the foundations of some systems of arithmetic, the characterization of the *methods* to verify certain properties are often essential. A claim of existence of certain objects is thus coupled with the presentation of a *concrete method* which produces these objects. That is, a statement of the form $\exists xA(x)$ be read: there exists a concrete method which produces an object x satisfying $A(x)$. $A(x)$ itself may in turn contain existential quantifiers, and hence such a statement be regarded as a nest of claims for desirable methods. Statements of this kind cannot be formalized in usual languages, but can only be characterized through cooperation of a formal system (in a usual language) in which certain reasonings are constrained. Due to the restrictions imposed upon the system in consideration, the existential quantifiers can be interpreted as claiming the existence of concrete methods.

The logic which underlies such a system in *intuitionistic*, or constructive. The central factor in the study of concrete mathematics is the interpretation of *implication;* that is, if $\exists X$and $\exists Y$ respectively claim existences of methods X and Y, then

$$\exists XA(X)\vdash\exists YB(Y)$$

should assert

$$\exists Z\forall X(A(X)\vdash B(Z(X))),$$

which should be read: there exists a method Z such that, for each method X verifying A, $Z(X)$ is a method to verify B. The universe of these methods varies according to the systems under consideration.

Here we consider a system of intuitionistic analysis, $\mathcal{S}(\boldsymbol{I})$, which is characterized by the bar induction as well as successive inductive definitions along some *accessible* orderings. $\mathcal{S}(\boldsymbol{I})$ is at the same time an abstraction of and a modest extension of the system ASOD in [3]. In ASOD, the formulas for in-

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