TSUKUBA J. MATH.
Vol. 12 No. 1 (1988), 269–272

ON THE EXISTENCE OF A STRAIGHT LINE

By

Fumiko OHTSUKA

§ 1. Introduction.

Let M be a connected, complete, non-compact, oriented and finitely connected Riemannian 2-manifold. The total curvature of such an M is defined to be an improper integral of the Gaussian curvature G with respect to the volume element of M and expressed as $C(M)=\int_{M}Gd_{M}$. The influence of total curvature of such an M have been investigated by many people. The pioneering work on total curvature was done by Cohn-Vossen in [1], which stated that if M admits total curvature, then $C(M)\leqq 2\pi\chi(M)$, where $\chi(M)$ is the Euler characteristic of M. He also proved in [2] that if a Riemannian plane M (i.e. M is a complete Riemannian manifold homeomorphic to $\boldsymbol{R}^{2}$) admits total curvature and if there exists a straight line on M, then $C(M)\leqq 0$. It is known that this is generalized as follows. (Confer section 4 in [4].); Let M have only one end. If such an M admits total curvature and if M contains a straight line, then $C(M)\leqq 2\pi(\chi(M)-1)$.

It is natural to consider whether the converse of the fact mentioned above is true or not. In this paper, we shall prove the following theorem.

THEOREM. *Let M be a connected, complete, non-compact, oriented and finitely connected Riemannian 2-manifold having one end. If M admits total curvature which is smaller than $2\pi(\chi(M)-1)$, then M contains a straight line.*

In the case where $C(M)=2\pi(\chi(M)-1)$, it is not always that M contains a straight line. In section 4, we shall show an example of a C^{2}-surface M whose total curvature is equal to 0 and on which there are no straight lines. Finally we shall note that if M has more than one end, then it is obvious that M contains a straight line.

§ 2. Preliminaries.

This section is devoted to introduce some definitions and the properties used throughout this paper.

Received June 22, 1987.