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ISOMETRIC IMMERSION OF RIEMANNIAN HOMOGENEOUS MANIFOLDS

By

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1. Introduction.

Bang-Yen Chen has introduced the notion of isometric immersion of finite type and proved that an equivariant isometric immersion of a compact Riemannian homogeneous manifold into a Euclidean space is of finite type [1].

In this paper we will prove the following theorem.

THEOREM. *Let M be a compact connected Riemannian homogeneous manifold with irreducible isotropy action. For an equivariant isametric immersion f of M into a Euclidean space E^N (considered as a Euclidean vector space) there exist a finite number of vector subspaces $E_0, E_1, \cdots, E_r$ of E^N, isometric immersions f_i of 1-type of M into E_i $(i=1, \cdots, r)$, constant vector v_0 in E_0 and positive constant $a_1, \cdots, a_r$ so that*

(1) $E^N=E_0+E_1+\cdots+E_r$ *(Euclidean direct sum)*

(2) $f=v_0+a_1f_1+\cdots+a_rf_r$.

REMARK. $a_1, \cdots, a_r$ satisfy $\sum_{i=1}^r a_i^2=1$.

2. Proof of Theorem.

Let M be a compact connected Riemannian homogeneous manifold with irreducible isotropy action. Let $G=I_0(M)$ be the identity component of the group of all isometries of M. G is a compact Lie group and acts on M transitively.

Let f be an equivariant isometric immersion of M into a Euclidean space E^N. Then there exists a Lie homomorphism ϕ of G into the isometry group $I(E^N)$ of E^N such that

$$f(g(p))=\phi(g)(f(p))$$

for any $g\in G$ and $p\in M$.

Since an isometric transformation of E^N is decomposed into a product of an orthogonal transformation and a parallel translation, we have a Lie homomorphism

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