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LARGE DEVIATION PRINCIPLE FOR DIFFUSION PROCESSES

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§1. Introduction.

Let $(\Omega, \boldsymbol{F}, P)$ be a probability space with an increasing family $\{\boldsymbol{F}_t; t\geqq 0\}$ of sub-σ-algebras of $\boldsymbol{F}$ and let $W(t)$ be a d-dimensional Brownian motion process adapted to $\boldsymbol{F}_t$. Then, we consider, on the Euclidean d-space R^d, the system of the stochastic differential equations;

$$dX^{\varepsilon}(t)=b(X^{\varepsilon}(t))dt+\varepsilon^{1/2}\sigma(X^{\varepsilon}(t))dW(t), \qquad X^{\varepsilon}(0)=x\in R^d, \tag{1.1}$$

where $\varepsilon>0$ is a small parameter, $b(x)=(b_i(x))_{i=1,\dots,d}$ is a d-vector function and $\sigma(x)=(\sigma_{ij}(x))_{i,j=1,\dots,d}$ is a $d\times d$-matrix function. Throughout the paper we assume that $b(x)$ and $\sigma(x)$ satisfy a local Lipschitz condition with respect to $x\in R^d$.

We shall study the behavior of $X^{\varepsilon}(t)$ as $\varepsilon\to 0$. This behavior will depend on the behavior of solutions of the dynamical system;

$$dX^{0}(t)=b(X^{0}(t))dt, \qquad X^{0}(0)=x\in R^d, \tag{1.2}$$

The system (1.1) can be considered as a small random perturbation of (1.2), with randomness expressed by a diffusion term $\varepsilon^{1/2}\sigma dW$. Set $a(x)=\sigma(x)\sigma^*(x)$, where the * means transpose. When $a(x)$ is uniformly elliptic and bounded, Freidlin and Wentzell [2] and also Friedman [3] obtain the large deviation principle for $X^{\varepsilon}(t)$. The former assumes the boundedness condition on $b(x)$ and $\sigma(x)$ together with a global Lipschitz condition in R^d. The latter assumes the boundedness condition on $a(x)$ and $b(x)$ together with a global Hölder condition with exponent $0<\alpha\leqq 1$ in the whole space R^d. Recently, under the positive definiteness condition on $a(x)$, Stroock [6] shows the large deviation principle, only assuming that $b(x)$ and $\sigma(x)$ satisfy a global Lipschitz condition in R^d.

The first purpose of this paper is to obtain a large deviation principle for $X^{\varepsilon}(t)$ under a satisfaction of some growth restriction on $b(x)$ and $\sigma(x)$. The most illustrative application is Theorem 3.1 with Example 2.1. It asserts that, in the problem of large deviations, the classical condition of linear growth in the phase variable of the coefficients can be weakened by allowing a logarithmic

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