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UNITARY-SYMMETRIC KÄHLERIAN MANIFOLDS AND POINTED BLASCHKE MANIFOLDS

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Introduction.

A unitary-symmetric Kählerian manifold is a Kählerian version of a rotationally symmetric (Riemannian) manifold (cf. Choi [3], Greene-Wu [5]). Precisely, a Kählerian manifold (M, g, J) of complex dimension n is unitary-symmetric at a point p of M if the linear isotropy group at p of the automorphism group of (M, g, J) is the unitary group $U(n)$. Of course, the complex space form is unitary-symmetric at every point.

The first purpose of this paper is to give one characterization of a connected, simply-connected, complete, unitary-symmetric Kählerian manifold. If M is compact, then the tangential cut locus C_p of p is spherical. Hence (M, g, J) is a Blaschke manifold at p and has a SL^p-structure (cf. Besse [1]). Then the second purpose is to give a sufficient condition in order that a connected, compact, unitary-symmetric Kählerian manifold has a SC^p-structure (Theorem D) (see Besse [1, p. 181]).

On the other hand, Greene-Wu [5, p. 85] introduced the notion of a Hermitian rotationally symmetric manifold of complex dimension 1 and Shiga [12] studied a Kählerian model, which is by defintion a Kählerian manifold with a pole p such that the linear isotropy group at p of the isometry group is $U(n)$. Note that their manifolds are unitary-symmetric Kählerian manifolds. The unitary-symmetric condition is a fairly strong one, because the result of Kaup [8, Folgerung 1.10] implies that a connected, unitary-symmetric Kählerian manifold is biholomorphic to one of the complex space forms. But there exist many complete unitary-symmetric Kählerian metrics, which are not isometric to them (see Mori-Watanabe [10]).

Throughout this paper, (M, g, J) is assumed to be a connected, complete Kählerian manifold of complex dimension $n \geqq 1$. To state our results, we prepapre the following. By Ω we denote the Kählerian form of (M, g, J). We frequently identify the tangent space $T_p(M)$ at a point p of M with the complex number n-space $\boldsymbol{C}^n$. Let $\exp_p$ be the exponential map of $T_p(M)$

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