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NATURAL TRANSFORMATIONS OF VECTOR FIELDS ON MANIFOLDS TO VECTOR FIELDS ON TANGENT BUNDLES

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There are well known classical examples of vector fields on the tangent bundle TM which can be constructed from a vector field on the base manifold M, namely the vertical lift and the complete lift. Furthermore, if we consider the tangent bundle over an affine manifold (M, ∇), we can define the horizontal lift of a vector field on (M, ∇) to TM. As we shall see in Section 1, the classical constructions are examples of "natural transformations of the second order".

We have two goals in this paper. The first is to describe explicitly *all* second order natural transformations of vector fields on manifolds into vector fields on their respective tangent bundles. The second is to describe explicitly *all pointwise* second order natural transformations of vector fields on manifolds with symmetric affine connections to vector fields on their respective tangent bundles.

As we have done in previous papers [1], [2], [7], we shall use for our purposes the concepts and methods developed by D. Krupka [3]-[5]. This leads to a system of partial differential equations to solve, and to the problem of geometric interpretation of all solutions. Our main results are formulated in Theorems 2.4 and 3.3.

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1. Classical lifts of vector fields to tangent bundles

In this paper we shall adopt the Einstein summation convention, unless otherwise stated. Also, we assume all manifolds and geometrical objects to be of class C^∞.

Let $(U; x^1, x^2, \cdots, x^n)$ and $(\bar{U}; \bar{x}^1, \bar{x}^2, \cdots, \bar{x}^n)$ be two systems of local coordinates in a smooth manifold M of dimension n such that the domain $U \cap \bar{U}$ is not

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