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TAME TWO-POINT ALGEBRAS

(Dedicated to Professor Tosiro Tsuzuku on his sixtieth's birthday)

By

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Introduction.

Throughout this paper, we will work over a fixed algebraically closed field k. Let Λ be a finite dimensional basic algebra. We may consider Λ as a locally bounded k-category. As well known, any locally bounded k-category Λ is given by a quiver with relations, that is, there is a locally finite quiver Q such that $\Lambda\cong kQ/I$, where kQ is the path-category and I is an ideal of kQ generated by linear combinations of paths of length $\geqq 2$ (see [3] for details). A module over a locally bounded k-category Λ is a k-linear functor from Λ to the category of k-vector spaces, nemely, a representation of the quiver satisfying the relations if Λ is given by a quiver with relations. We will denote by mod Λ the category of all finite dimensional left Λ-modules.

In the present paper, we are interested in two-point algebras, namely, algebras which have just two non-isomorphic simple left modules. Our aim is to classify two-point algebras of certain classes according to their representation types. An algebra Λ is said to be representation-finite if there are only a finite number of pairwise non-isomorphic indecomposable objects in mod Λ, to be wild if there is an exact embedding mod $k\Omega\rightarrow$mod Λ, where $k\Omega$ is the path-algebra of the quiver $\Omega: \circlearrowleft\cdot\circlearrowright$, which is a representation equivalence with the corresponding full subcategory of mod Λ, and to be tame if Λ is neither representation-finite nor wild. There has been given the complete list of the maximal representation-finite two-point algebras [3].

Covering techniques ([1], [3], [5] and [6]) will play an indispensable role in deciding the representation type of a given algebra. For a certain class of algebras, by taking appropriate Galois coverings, the problem can be reduced to the calculation of vector space categories, which have been classified in [12] (see also [9]). On the other hand, we will come across an algebra which can be obtained as a quotient of a suitable Galois covering of the tame local algebra $\tau\circlearrowleft\cdot\circlearrowright\sigma$ with $\sigma^2=\tau^2=0$ [11], thus is tame. The similar argument will also apply to the situation that there is a Galois covering of a given algebra which has a wild algebra as a finite quotient.

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