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PSEUDO-DIFFERENTIAL OPERATORS ON BESOV SPACES

By

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Introduction.

In the present paper, we shall study the pseudo-differential operators on Besov spaces $B_{p,q}^{s}$ ($s\in\boldsymbol{R}$, p, $q\in[1, \infty]$), and give systematical boundedness theorems for pseudo-differential operators whose symbols belong to the Hörmander class $S_{\rho,\delta}^{m}$ ($m\in\boldsymbol{R}$, ρ, $\delta\in[0, 1]$). Besov spaces $B_{p,q}^{s}$ are generalization of both Hölder spaces C^{s} and Sobolev spaces H_{2}^{s} (Remark 1.1).

It has already been known that symbols belonging to the class $S_{1,\delta}^{0}$ generate $B_{p,q}^{s}$-bounded ($s>0$ if $\delta=1$) pseudo-differential operators. See, for example, Gibbons [8] ($S_{1,0}^{0}$) and Bourdaud [1] ($S_{1,\delta}^{0}$). Our primary object is to show the same result for the general class $S_{\rho,\delta}^{m}$.

On the other hand, in order for symbols $\sigma(x, \xi)$ to generate $B_{p,q}^{s}$-bounded pseudo-differential operators, $\sigma(x, \xi)$ need not be so regular but need only small regularity of Besov or Hölder spaces type depending on $B_{p,q}^{s}$. This fact is verified if we consider pointwise multipliers $\sigma(x)$ which are special cases of pseudo-differential operators; cf. Triebel [19], Section 2.8. Gibbons [8] and Bourdaud [1] also considered non-regular symbols satisfying Besov spaces type estimates, and gave boundedness theorems for such symbols. Our secondary object is to discuss to what degree we can relax regularity conditions for symbols.

In order to carry out our two objects, we shall define new symbol classes $S_{\rho,\delta}^{m}(B_{(p,p'),(q,q')}^{(\lambda,\lambda')})$ on $\boldsymbol{R}_{x}^{n}\times\boldsymbol{R}_{\xi}^{n}$ which are generalization of the Hörmander class $S_{\rho,\delta}^{m}$ (Definition 3.3). These classes consist of non-regular symbols $\sigma(x, \xi)$ which have only $B_{p,q}^{\lambda}$ (resp. $B_{p',q'}^{\lambda'}$)-regularity with respect to the variable x (resp. ξ). Our main result is the following (Theorem 4.1).

MAIN THEOREM. *Let p, $q\in[1, \infty]$, $s\in\boldsymbol{R}$, ρ, $\delta\in[0, 1]$, and let $s>0$ in case of $\delta=1$. If $\lambda>\mu(p, s)$, pseudo-differential operators on $\boldsymbol{R}^{n}$ with symbols belonging to the class $S_{\rho,\delta}^{m(p)}(B_{(\infty,\infty),(\infty,1)}^{(\lambda,\bar{h}(p))})$ are bounded on $B_{p,q}^{s}$. Here we have used the following notations:*

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