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REDUCTION OF THE CODIMENSION OF TOTALLY REAL SUBMANIFOLDS OF A COMPLEX SPACE FORM

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Iutroduction.

A submanifold M of a Kaehlerian manifold $\bar{M}$ is said to be *totally real* if each tangent space to M is mapped to the normal space by the complex structure of $\bar{M}$. Many subjects for totally real submanifolds were studied from various different points of view, as ones of which Naitoh [9] and Naitoh and Takeuchi [10] classified an n-dimensional totally real submanifold with parallel second fundamental form in $\boldsymbol{P}_n\boldsymbol{C}$, and Ohnita [11] and Urbano [16] showed recently that the second fundamental form of such a submanifold of non-negative curvature is parallel, independently. Besides, the study for 3-dimensional totally real submanifols of S^6 by Mashimo [8] is also interesting.

In this paper the reduction of Allendoerfer type for the codimension of totally real submanifolds of a complex space form is treated with. As for all sorts of studies mentioned above, it is important that the dimension of the submanifolds is half of that of the ambient space. The purpose of this article is to show that the fact is essential, namely, to verify the following

THEOREM. *Let $\bar{M}$ be a complex space form of complex dimension m, and M an n-dimensional totally real submanifold of $\bar{M}$. If the induced f-structure in the normal bundle is parallel, then there exists a totally geodesic comlex space form M_0 of complex dimension n of $\bar{M}$ in which M is totally real.*

In the first section, preliminaries about totally real submanifolds of a complex space form are prepared for and the theorem is proved in the case where the ambient space is complex Euclidean. In § 2, a $(2m+1)$-dimensional anti-de Sitter space H_1^{2m+1} and the Sasakian structure on such a manifold are recalled. Lorentzian submanifolds of H_1^{2m+1} are treated with in the next section and the theorem is proved in § 4 provided that the ambient space is hyperbolic. In the last section the proof in $\boldsymbol{P}_m\boldsymbol{C}$ will be sketched.

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