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A CHARACTERIZATION OF CLOSED s-IMAGES OF METRIC SPACES

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Throughout the present note, we assume that all spaces are regular topological spaces and all mappings are continuous. Let N denote the set of all natural numbers.

Recall from [7] a collection $\mathcal{P}$ of subsets of a space X is called a *k-network* for X if for every compact subset K of X and every open set U of X with $K\subset U$, there is a finite subcollection $\mathcal{P}'$ of $\mathcal{P}$ such that $K\subset \cup\{P: P\in\mathcal{P}'\}\subset U$. A collection $\mathcal{P}$ of subsets of a space X is called a *cs-network* for X if for every sequence $\{x_n: n\in N\}$ converging to a point $x\in X$ and every neighborhood U of x, there is an element $P\in\mathcal{P}$ such that $P\subset U$ and $\{x_n: n\in N\}$ is eventually in P ([4]). A space is said to be an $\aleph$-space if it has a σ-locally finite k-network ([6]). A mapping f from a space X to a space Y is called an *s-mapping* if $f^{-1}(y)$ has a countable base for each $y\in Y$.

Recently, L. Foged [2] proved an interesting characterization of Lašnev spaces: A space X is Lašnev space (i. e. X is a closed image of a metric space) if and only if X is a Fréchet space with a σ-hereditarily closure preserving k-network. On the other hand, Y. Tanaka showed that every closed s-image X of a metric space is an $\aleph$-space if any closed metrizable subset of X is locally compact ([9, Lemma 4.1]). (Using this result, he gave a characterization for the product space $X\times Y$ of closed s-images X and Y of metric spaces to be a k-space (see [9, Theorem 4.3]).) He asked in the same paper whether every closed s-image of a metric space is an $\aleph$-space. The purpose of this note is to answer the above question and simultaneously to get a characterization of Fréchet $\aleph$-spaces.

Our result is the following.

THEOREM. *For a regular space X, the following are equivalent.*

(a) *X is a Fréchet $\aleph$-space.*

(b) *X is a closed s-image of a metric space.*

(c) *X is a Fréchet space with a point countable, σ-closure preserving,*

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