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BENFORD'S LAW FOR LINEAR RECURRENCE SEQUENCES

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1. Introduction.

One of the authors of the present paper, Kenji Nagasaka, considered, in his preceding article [4], various sampling procedures from the set of all positive integers and examined for the resulting sampled integers whether Benford's law holds or not.

J. L. Brown, Jr. and R. L. Duncan [1] treated linear recurrence sequences and proved that, under several conditions on the corresponding characteristic equations, Benford's law is valid for certain linear recurrence integer sequences. It was shown by Lauwerens Kuipers and Jau-Shyong Shiue [3] that this result was able to be established by using one of J. G. van der Corput's difference theorems [5], [6].

Nagasaka succeeded in generalizing the main theorem of Duncan and Brown, which is Theorem 4.3 in [4]. Detailed study of linear recurrence sequences, especially of order 2 is made in Theorem 4.1 and Theorem 4.2. But it still remains several cases ignored.

In this joint paper, we shall adopt one of van der Corput's difference theorems as a main tool and prove some results on Benford's law for linear recurrence sequences.

In the next Section, recurrence sequences of order 1 will be considered and we shall show sufficient conditions for Benford's law to be valid, which contain Theorem 3.2 in [4] as a special case.

In Section 3 we shall give proofs of Theorem 4.1 and Theorem 4.2 in [4] based upon one of van der Corput's difference theorems. These Theorems do not contain the case where the corresponding characteristic equation has two complex conjugate roots. We shall show further that Benford's law holds for linear recurrence sequences when their corresponding characteristic equations have two purely imaginary conjugate roots.

In the final Section, we shall consider general linear recurrence sequences of arbitrary order and prove analogous results as in the case of order 2.

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