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ON THE THEORY OF MULTIVALENT FUNCTIONS

By

Mamoru NUNOKAWA

I would like to dedicate this paper to the late Professor Shigeo Ozaki.

1. Introduction.

Let $A(p)$ be the class of functions of the form

$$f(z)=\sum_{n=p}^{\infty}a_{n}z^{n} \qquad (a_{p}\neq 0\,;\ p\in N=\{1,2,3,\cdots\}) \tag{1}$$

which are regular in $|z|<1$.

A function $f(z)$ in $A(p)$ is said to be p-valently starlike iff

$$\mathrm{Re}\frac{zf'(z)}{f(z)}>0 \qquad (|z|<1).$$

We denote by $S(p)$ the subclass of $A(p)$ consisting of functions which are p-valently starlike in $|z|<1$.

Further, a function $f(z)$ in $A(p)$ is said to be p-valently convex iff

$$1+\mathrm{Re}\frac{zf''(z)}{f'(z)}>0 \qquad (|z|<1).$$

Also we denote by $C(p)$ the subclass of $A(p)$ consisting of all p-valently convex functions in $|z|<1$.

2. Preliminaries.

At first, we prove the following lemma by using the method of Ozaki [10].

LEMMA 1. *Let* $f(z)\in A(p)$ *and*

$$\mathrm{Re}\frac{zf'(z)}{f(z)}>K \qquad in \quad |z|<1 \tag{2}$$

where K *is a real bounded constant, then we have*

$$f(z)\neq 0 \qquad \text{in} \quad 0<|z|<1.$$

PROOF. Suppose that $f(z)$ has a zero of order n $(n\geqq 1)$ at a point α that satisfies $0<|\alpha|<1$. Then $f(z)$ can be written as $f(z)=(z-\alpha)^{n}g(z)$, $g(\alpha)\neq 0$ and

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