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THE DIFFERENCES BETWEEN CONSECUTIVE ALMOST-PRIMES

By

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1. Introduction.

In 1940 P. Erdös [1] proposed the problem to estimate the sum

$$D(x)=\sum_{p_n\leqq x}(p_{n+1}-p_n)^2$$

where p_n denotes the n-th prime. A. Selberg [10] and D. R. Heath-Brown [4] proved that

$$D(x)\ll x(\log x)^3$$

under the Riemann hypothesis, and that, for any $\varepsilon>0$,

$$D(x)\ll x^{7/6+\varepsilon}$$

under the Lindelöf hypothesis, respectively. Furthermore, Heath-Brown [5] showed unconditionaly that, for any $\varepsilon>0$,

$$D(x)\ll x^{23/18+\varepsilon},$$

and he [6] conjectured that

$$D(x)\sim 2x(\log x) \quad \text{as } x\to\infty.$$

U. Meyer considered in his Dissertation the almost-prime analogy of $D(x)$. Let P_2 denote the set of integers with at most two prime factors, multiple factors being counted multiplicity. We replace the primes in $D(x)$ by the almost-primes P_2, and denote the resulting sum $D_2(x)$. In [8] he showed, by the weighted version of a zero density estimate for the Riemann zeta-function, that

$$D_2(x)\ll x^{1.285}(\log x)^{10}.$$

It is the purpose of this paper to make an improvement upon this upper bound.

THEOREM. *We have*

$$D_2(x)\ll x^{1.023}$$

where the implied constant is effectively computable.

In contrast to the Meyer's argument, we appeal to sieve methods, which are the weighted linear sieve of Greaves' type [3] and the prototype of an additive

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