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SPECTRAL THEORY FOR SYMMETRIC SYSTEMS IN AN EXTERIOR DOMAIN

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1. Introduction

The present work is devoted to the investigation of spectral problems of a first order symmetric system in an exterior domain Ω of $\boldsymbol{R}^n$ of the form

$$Hu=H(x, D)u=\sum_{j=1}^{n}A_j(x)D_ju+C(x)u, x\in\Omega \tag{1.1}$$

where $D_j=-i\partial_j, \partial_j=\partial/\partial x_j, 1\leq j\leq n, u=u(x)={}^t(u_1(x), \cdots, u_d(x))$ is a $\boldsymbol{C}^d$-valued function and $A_j(x), 1\leq j\leq n, C(x)$ are $d\times d$ matrix valued functions.

The spectral theory for symmetric systems in the whole space $\boldsymbol{R}^n$ has been extensively investigated by many authors under various conditions (see Schulenberger-Wilcox [9], Tamura [11], Weder [12], and their references). On the other hand there are not so many works treating exterior boundary value problems for the system (1.1) (Lax-Phillips [3], Schmidt [8], and Stefanov-Georgiev [10] by the time dependent method, and Kikuchi [1] and Mochizuki [5] by the stationary method). In all of them the coefficients are assumed to take constant values outside bounded balls and further, restrictive conditions are imposed on the geometrical structure of the slowness surfaces of the free systems except [10].

In this paper we shall employ the commutator method due to Mourre [6] to avoid difficulties derived from the slowness surface and the formulation of radiation conditions, and to prove the limiting absorption principle for the long range perturbed system (1.1). To this end we restrict ourselves to work in the domain Ω satisfying the following: The domain Ω lies in the exterior of its boundary $\partial\Omega$ which is a smooth and compact hypersurface enclosing the origin. Further, there exists a positive $C^\infty(\boldsymbol{R}^n\backslash\{0\})$-function $\rho(x)$, positively homogeneous of degree zero such that

$$|x|=\rho(x) \text{ if and only if } x\in\partial\Omega. \tag{1.2}$$

Consider the following boundary value problem with a spectral parameter $z\in\boldsymbol{C}$:

$$(H(x, D)-z)u(x)=f(x) \text{ in } \Omega,$$

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