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A NOTE ON STRONG HOMOLOGY OF INVERSE SYSTEMS

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1. Introduction.

Ju. T. Lisica and the author have defined in [4] strong homology groups $H_p(\boldsymbol{X}; G)$ of inverse systems of spaces $\boldsymbol{X}=(X_\lambda, p_{\lambda\lambda'}, \Lambda)$ over directed cofinite sets Λ (every element $\lambda\in\Lambda$ has only finitely many predecessors). It was shown in [5] that these groups are functors on the coherent prohomotopy category CPHTop, introduced in [2] and [3]. The notion of strong or Steenrod homology $H_p^s(X; G)$ of an arbitrary space X was then defined [1], [6] and shown to be a functor on the strong shape category SSh [2], [3]. The procedure consisted in choosing a cofinite ANR-resolution $\boldsymbol{p}: X\rightarrow\boldsymbol{X}$ of X ([7], [8], [9]) and of defining $H_p^s(X; G)$ as $H_p(\boldsymbol{X}; G)$. That the group $H_p^s(X; G)$ does not depend on the choice of the resolution is a consequence of the following factorization theorem ([3], Theorem II. 2.3). If $\boldsymbol{p}: X\rightarrow\boldsymbol{X}$ is a resolution and $\boldsymbol{f}: X\rightarrow\boldsymbol{Y}$ is a coherent map into a cofinite ANR-system, then there exists a unique coherent homotopy class of coherent maps $\boldsymbol{g}: \boldsymbol{X}\rightarrow\boldsymbol{Y}$ such that $\boldsymbol{g}\boldsymbol{p}$ and $\boldsymbol{f}$ are coherently homotopic.

The definition of composition in CPHTop and the proof of the factorization theorem essentially used the assumption that the index sets Λ be cofinite. On the other hand, the construction of the homology groups $H_p(\boldsymbol{X}; G)$ did not require this assumption. Therefore, it remained unclear whether one can use also non-cofinite ANR-resolutions to determine the homology groups $H_p^s(X; G)$ of the space X. To prove that this is indeed the case is the main purpose of this paper. Such an information can prove useful in situations where a non-cofinite ANR-resolution naturally arises.

The main idea of the proof is to replace a given ANR-resolution $\boldsymbol{p}: X\rightarrow\boldsymbol{X}$ by a cofinite ANR-resolution $\boldsymbol{p}^*: X\rightarrow\boldsymbol{X}^*$ using the "trick" described in ([9], Theorem I, 1.2). What remains to be done is to exhibit a natural isomorphism $u_*: H_p(\boldsymbol{X}; G)\rightarrow H_p(\boldsymbol{X}^*; G)$. The correct formula for u_* is easily found. However, the formula for the inverse v_* of u_* is less obvious. Even more complicated is the verification of the two equalities $u_*v_*=1$, $v_*u_*=1$.

In order to simplify notations throughout the paper we omit the coefficient groups G, although all results hold for an arbitrary G.

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